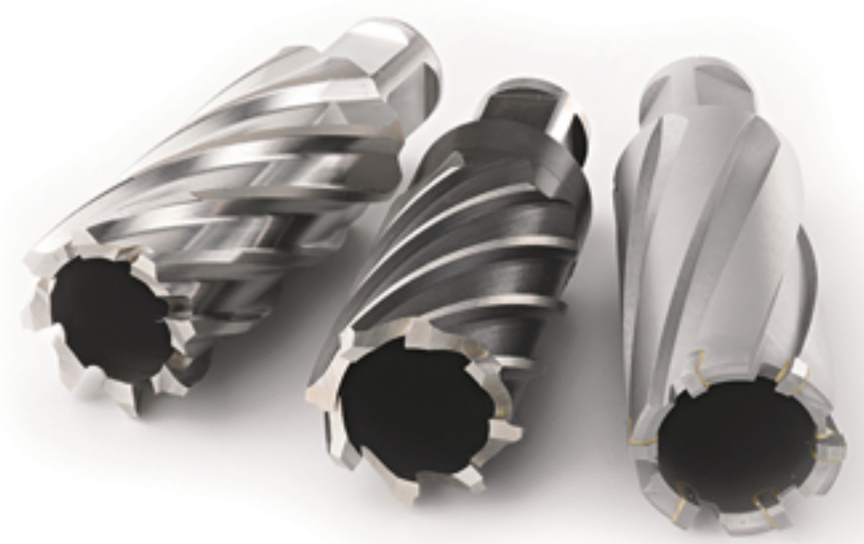
Introduction to Computational solid mechanics

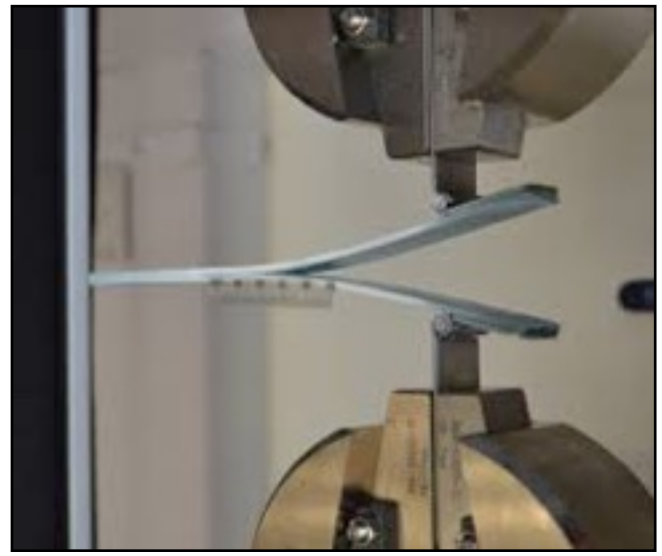
Eva Casoni

Why do we study solid mechanics?

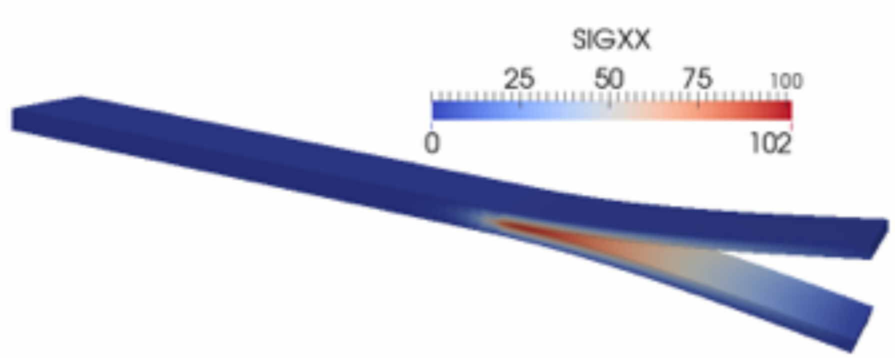
Anyone concerned with the strength and physical performance of natural/man-made structures should study Mechanics of materials

From small pieces…

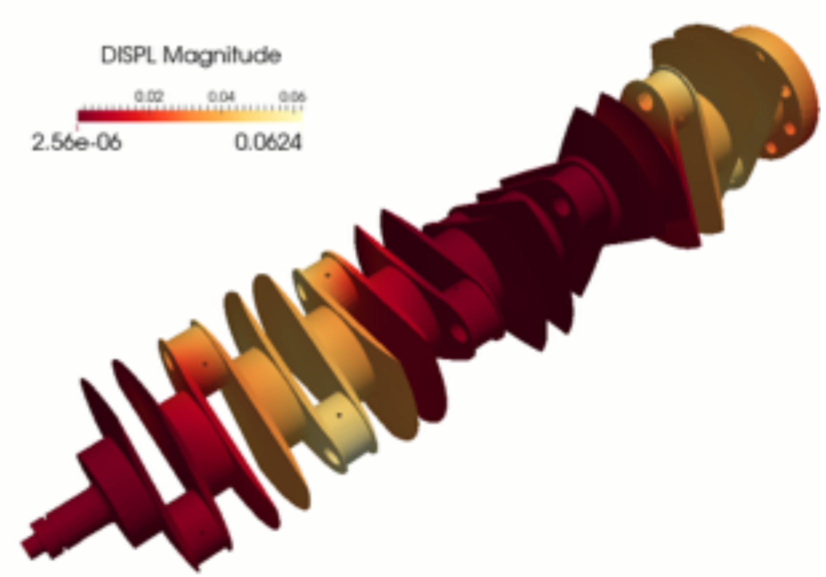




drill

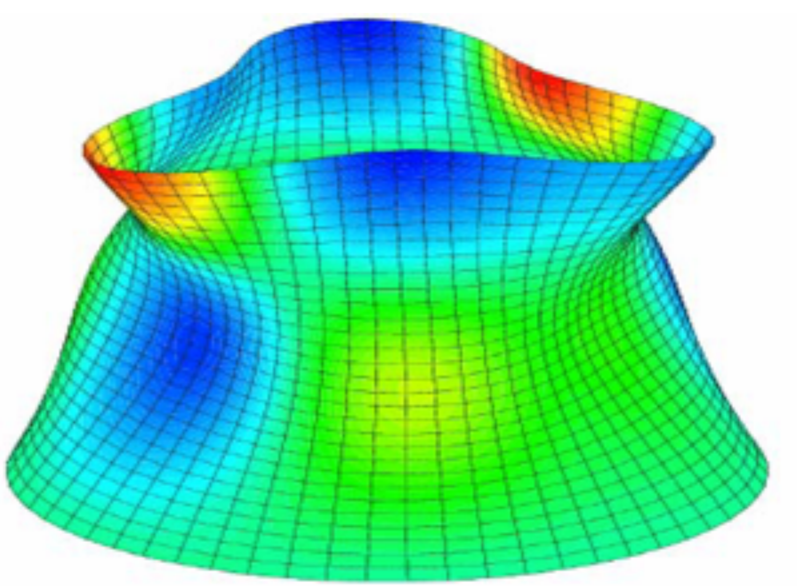


double cantilever beam

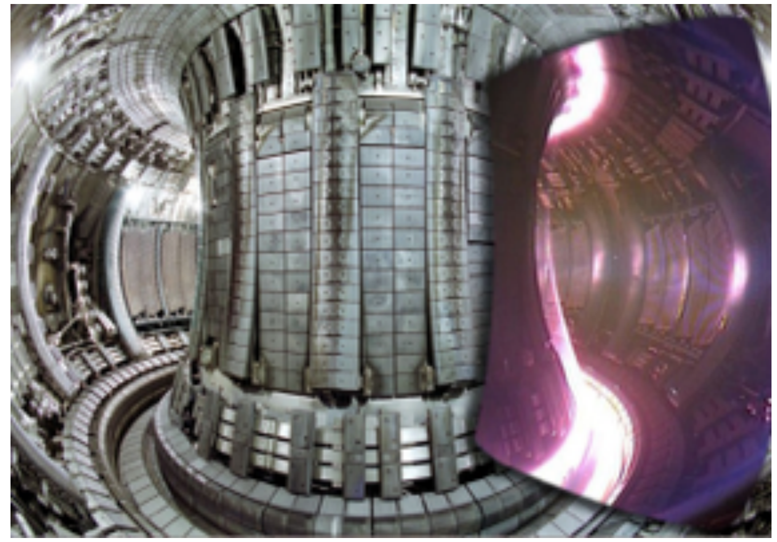


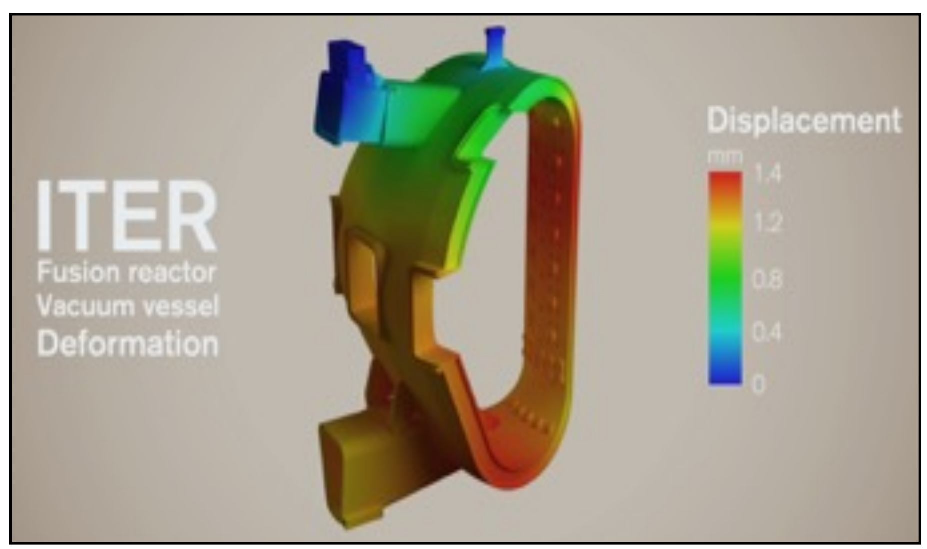
crankshaft

… to big structures

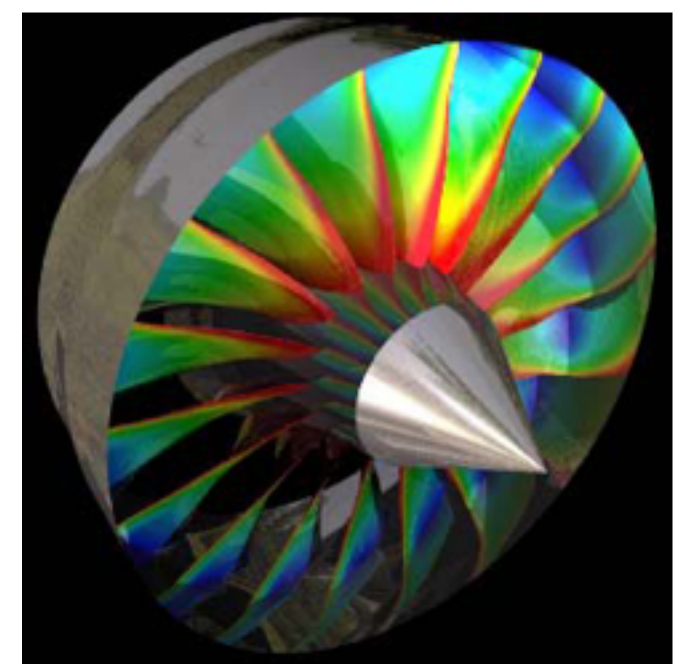


Thermo-mechanical simulation of a refrigeration tower



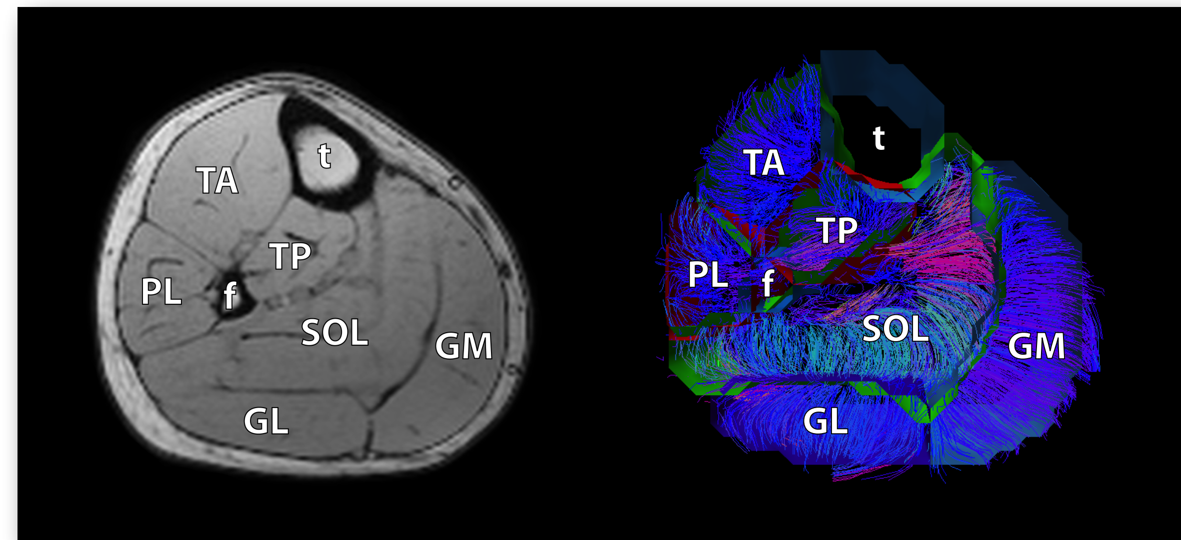


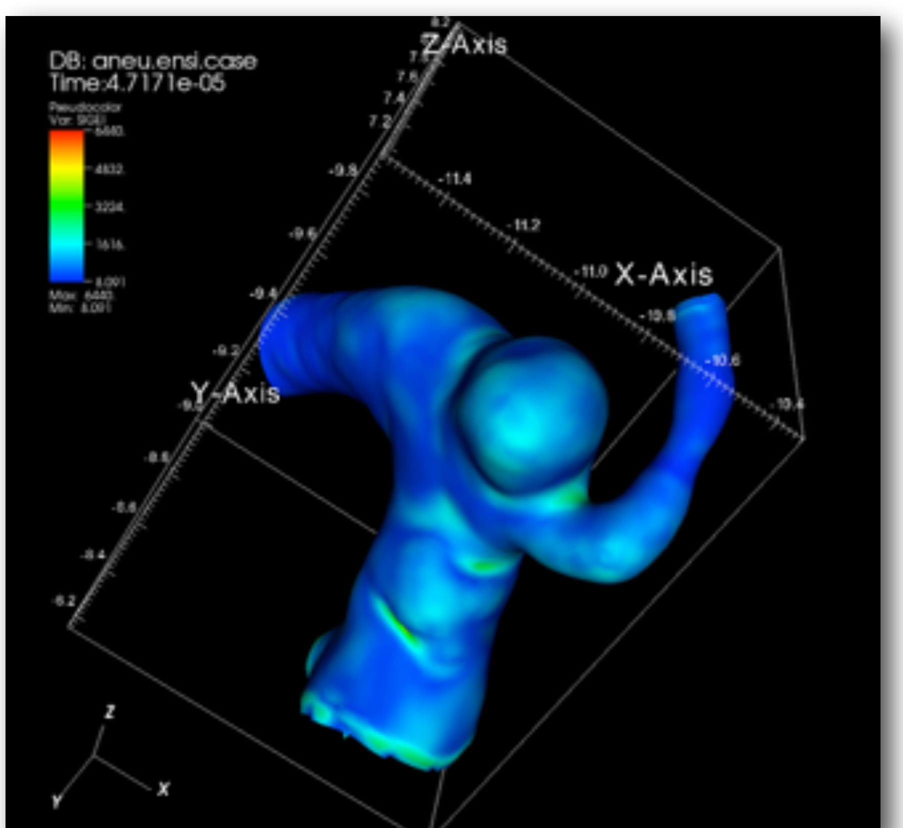
Seismical structural analysis of the main torus of Tokamak fusion reactor



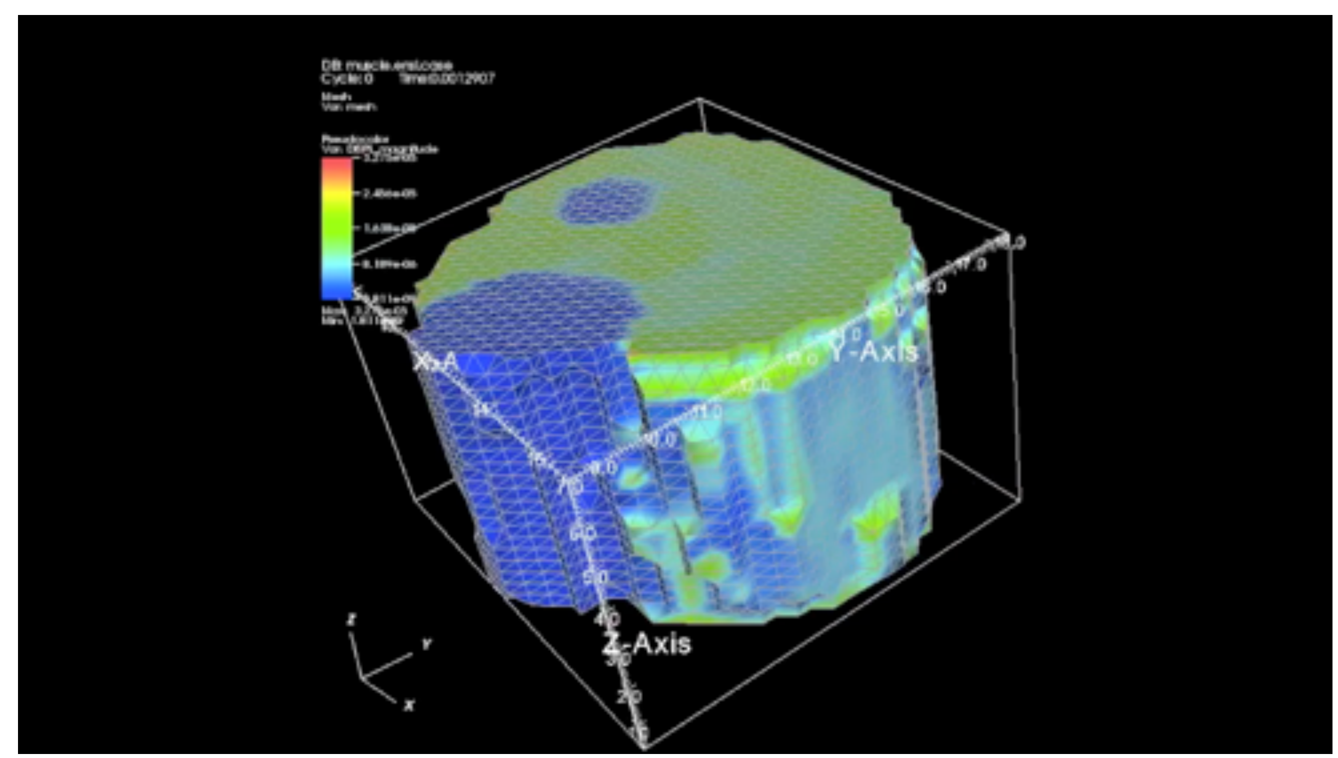
Thermo-hydro-mechanics simulation of a Rolls Royce turbine of a plane

… and not only structures…



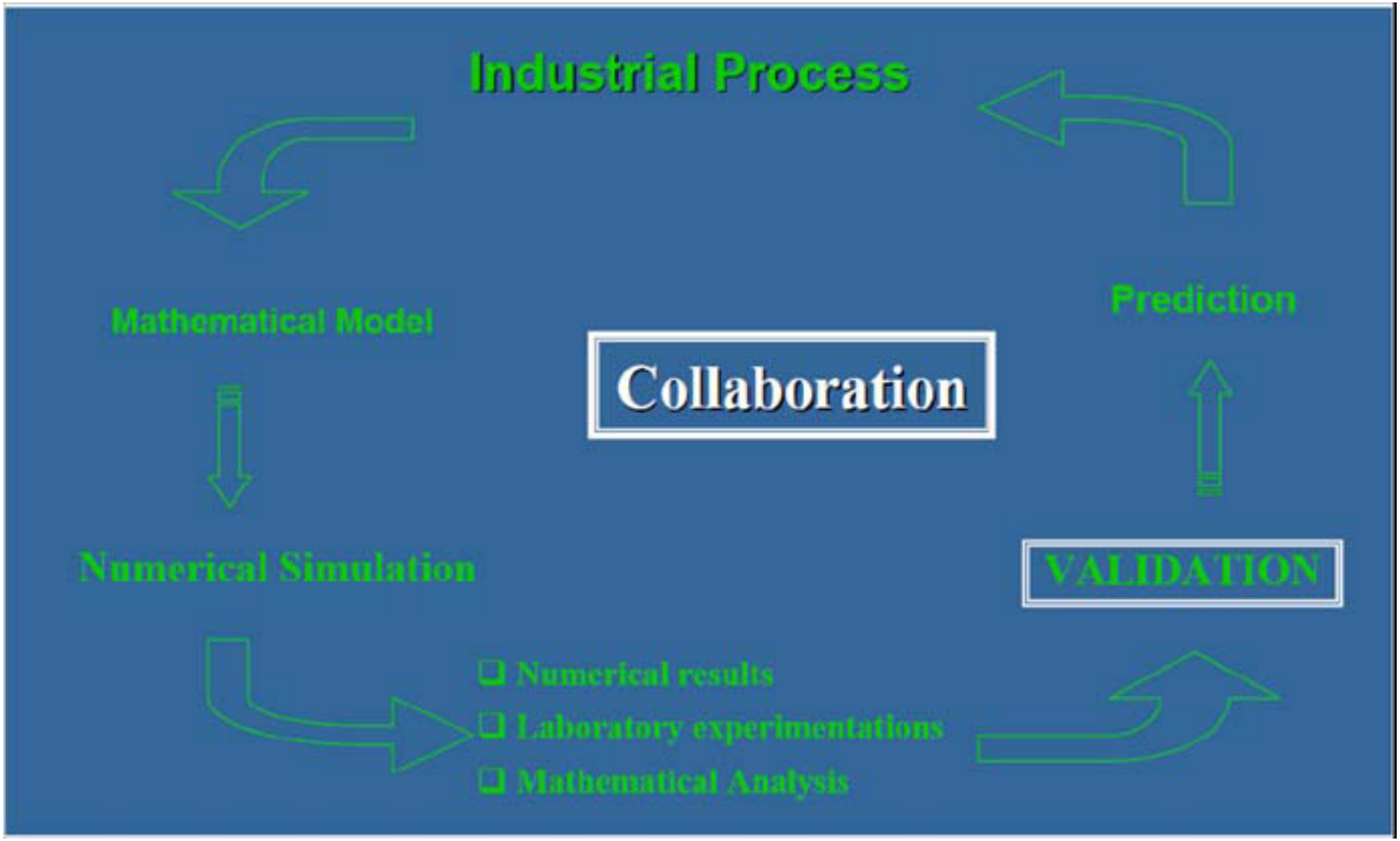


Cerebral aneurysms *(in collaboration with George Mason University)*



Long skeletal muscles *(in collaboration with J. Georgiadis, University of Illinois)*

Numerical simulation

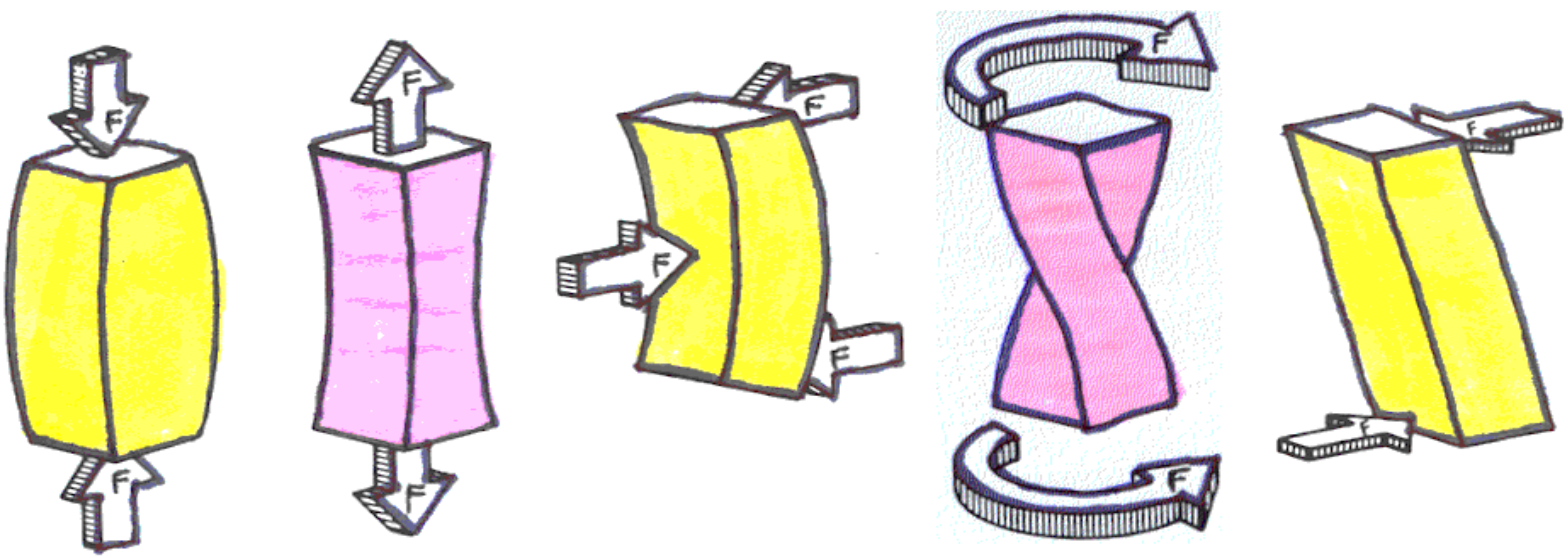


Outline

- Mechanics of materials
- Basics on continuum solid mechanics
  - Kinematics
  - Balance of momentum equation
  - Principle of Virtual work and weak form
- Elasticity
- Hyperelasticity
- Composite materials
  - Damage and failure
- Fracture
  - XFEM
  - CZM

Mechanics of materials

Mechanics of materials, solid mechanics, is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading.



Compression *Tension (stretched)* *Bending* *Torsion (twisted)* *Shearing*

Composite materials

Material types

ISOTROPIC - same material properties in all directions, steel is a typical example.

Easy to measure properties

ANISOTROPIC - different material properties in all directions, a chunk of volcanic rock is an example.

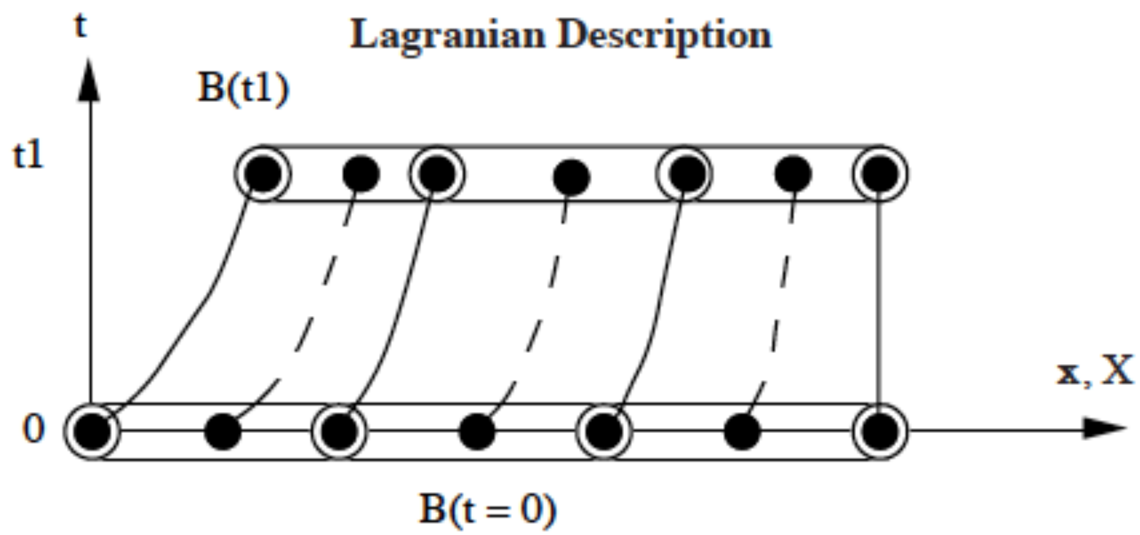
Tough to measure or predict properties

ORTHOTROPIC - special case of anisotropic, clear material directionality in 3 directions, wood is an example (material properties in three perpendicular directions -axial, radial and circumferential- are different).

Measurable and predictable properties - some challenges

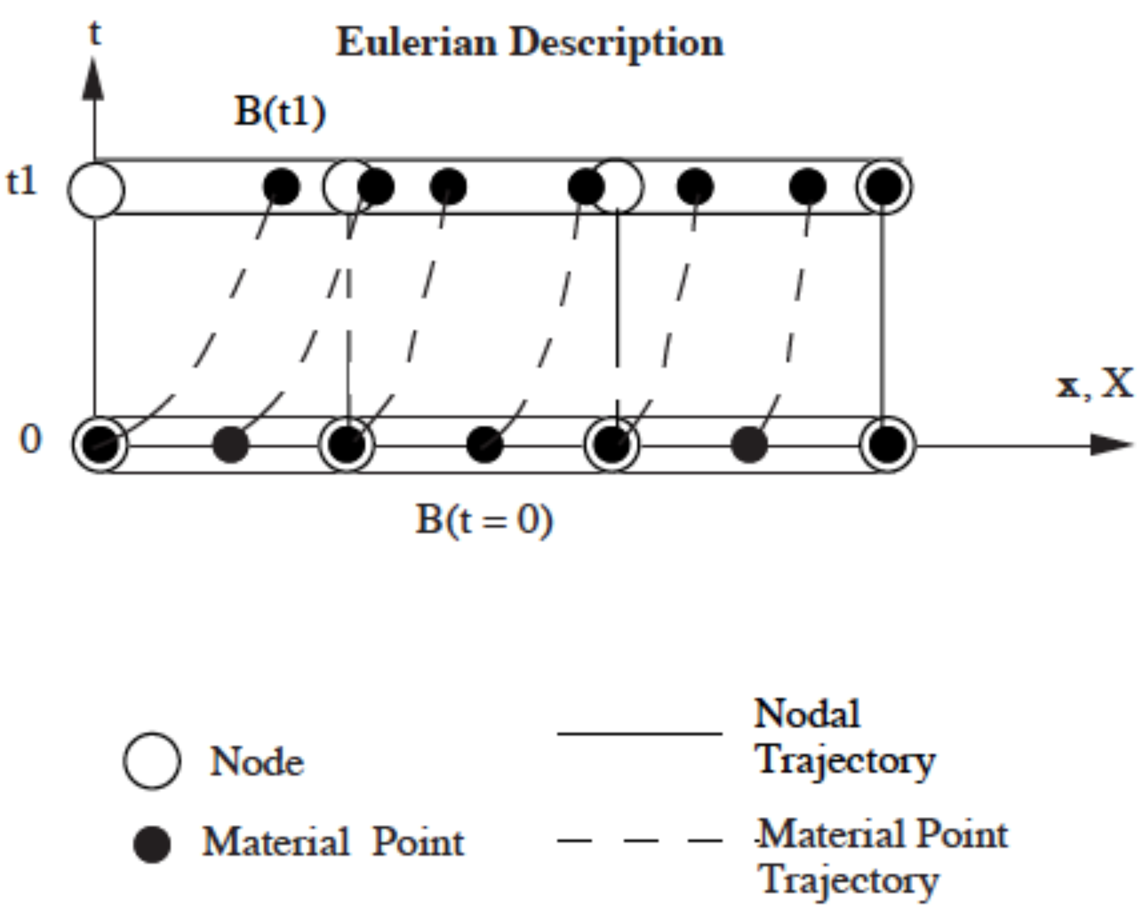
Basics on continuum solid mechanics

Lagrangian vs. Eulerian description



Lagrangian or material description

- Physical quantities (and equations) are described with material coordinates X.
- For example, M(X,t) represents the physical quantity (temperature, stress, etc.) of the particle at t=0 was at position X, and a function of time.
- If we fix X, we follow the same material particle at different points of the space.

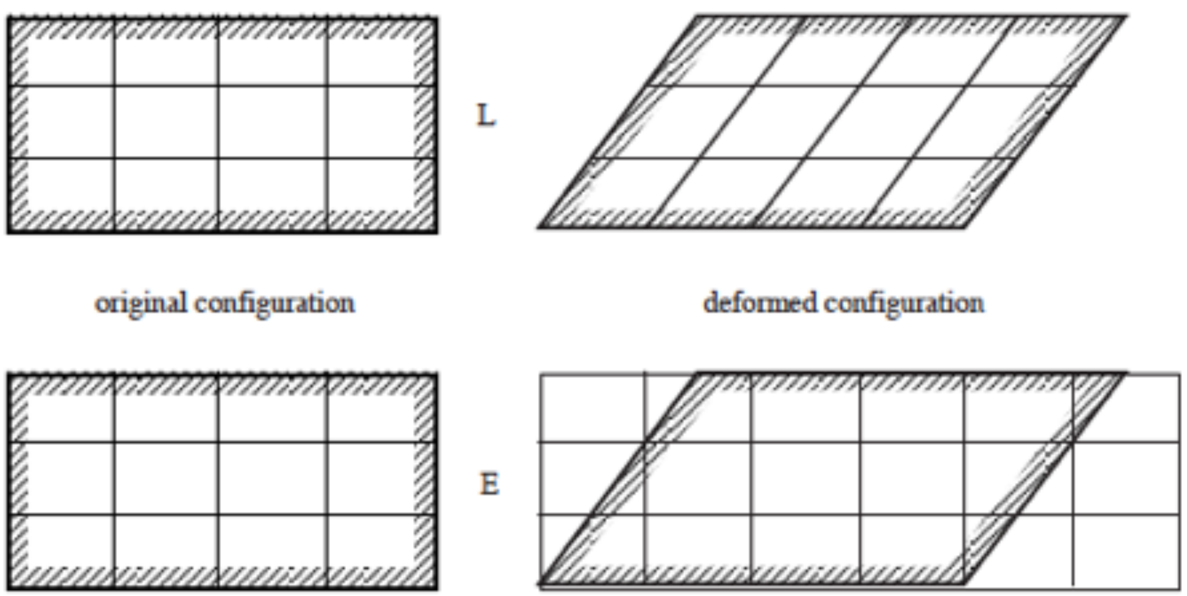


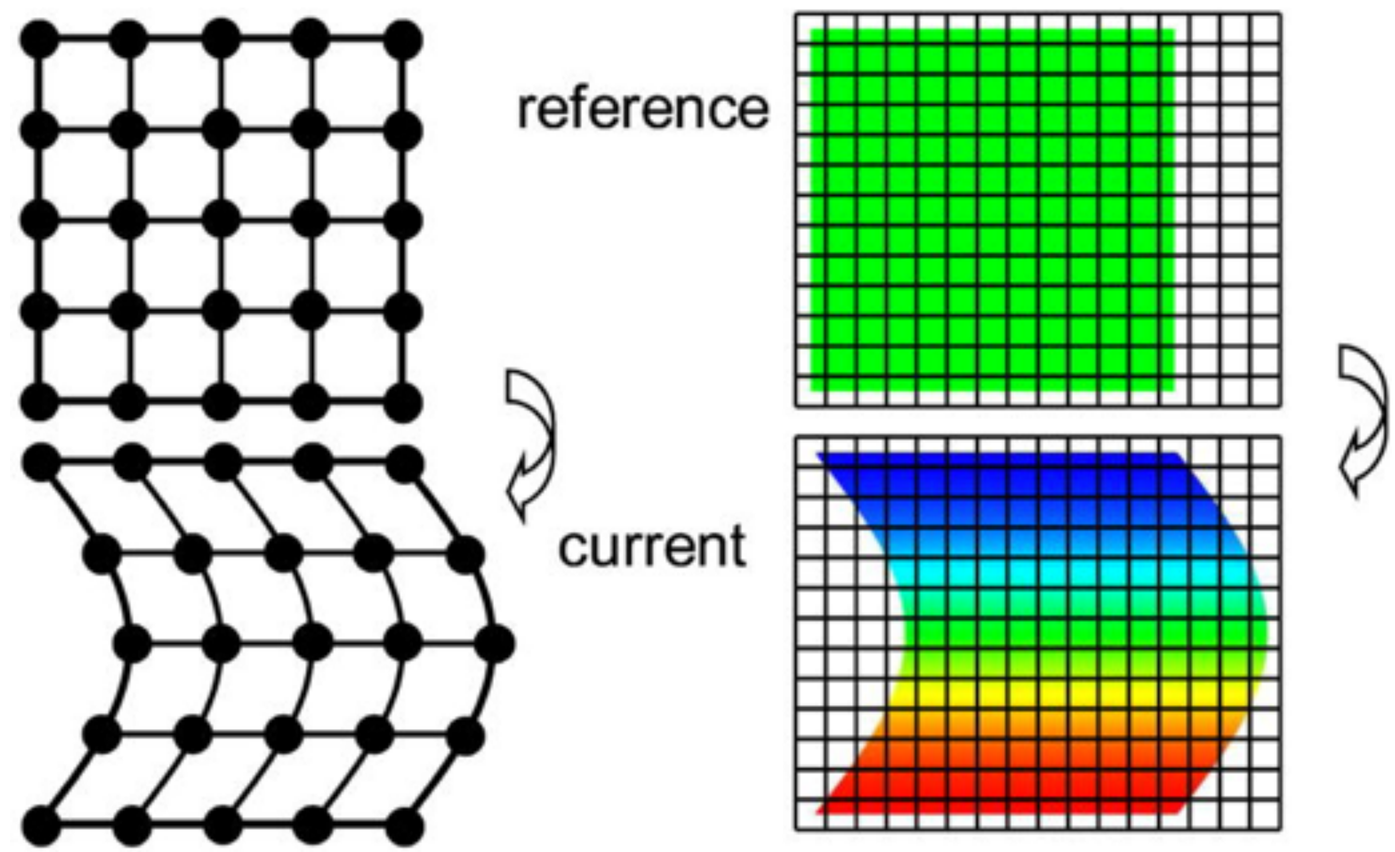
Eulerian or spatial description

- Physical quantities (and equations) are described with spatial coordinates x.
- For example, m(x,t) represents the physical quantity (temperature, stress, etc.) at time t at the spatial position x.
- If we fix x, we are observing the same spatial points, where different material particles may be located at different times. .

Lagrangian vs. Eulerian description

The motion is given by the deformation mapping $\boldsymbol{x} = \phi(\boldsymbol{X}, t)$





(a) Lagrangian approach (b) Present Eulerian approach

We will use Lagrangian formulation

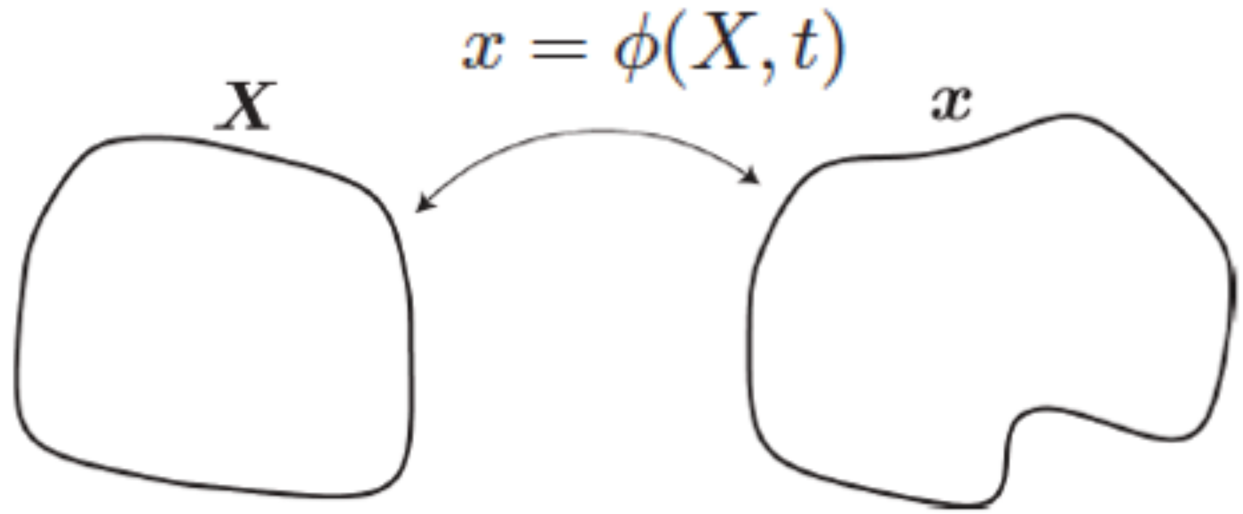
Continuum solid mechanics: frame of reference

- Motion and deformation (kinematics)
- Useful measures in non-linear continuum mechanics (depend on the configuration)
  - Strain measures: Green-Lagrange, rate of deformation, …
  - Stress measures: Cauchy, nominal stress, 1PK, 2PK, …

    Large variety of measures! (adding nothing fundamental…)
- Conservation equations
  - Common to solids and fluids: mass, momentum, energy
  - Momentum equation $\longrightarrow$ equilibrium equation
- Large deformations vs. small deformations

Continuum solid mechanics: frame of reference

Map from reference domain to physical domain



$$\boldsymbol{F} := \frac{\partial x}{\partial X} \qquad J := \det \boldsymbol{F} \qquad v(X,t) := \frac{\partial x}{\partial t}$$

Conservation of linear momentum equation

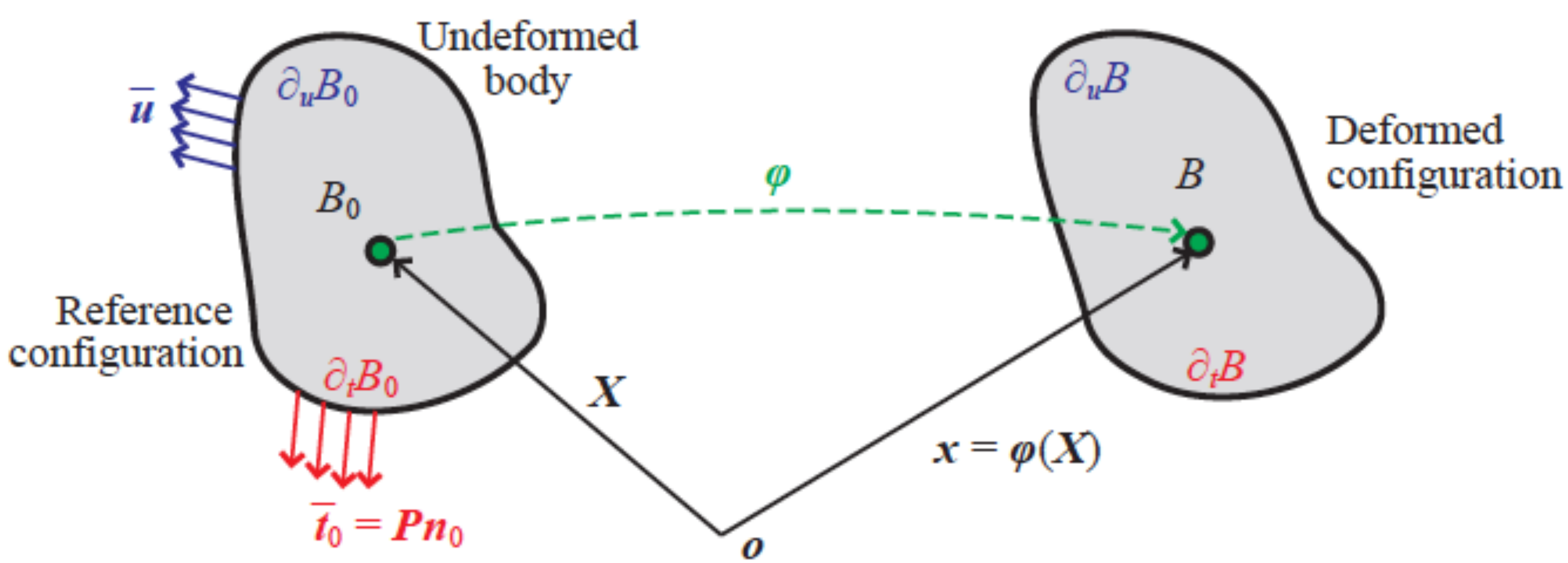
$$\rho_0 \frac{\partial \mathbf{v}(\boldsymbol{X},t)}{\partial t} = \nabla \cdot \mathrm{P} + \rho_0 \boldsymbol{b}$$

using first Piola-Kirchhoff ***P***.

Deformation mapping

- Initial domain B_0 is deformed to B: **mapping** from B_0 to B
- $\mathbf{X}$: material point in B_0 $\qquad$ $\mathbf{x}$: material point in B
- Infinitesimal length $d\mathbf{X}$ in B_0 deforms to $d\mathbf{x}$ in B
- Remember that the mapping is **continuously differentiable**

$$\mathbf{x} = \mathbf{X} + \mathbf{u} \quad \Longrightarrow \quad \mathbf{x} = \varphi(\mathbf{X},t) = \mathbf{X} + \mathbf{u}(\mathbf{X},t)$$



Stress and strain measures

Useful quantities

- Deformation gradient tensor (gradient of the mapping)

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \qquad F_{ij} = \frac{\partial x_i}{\partial X_j} \qquad F = \begin{pmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{pmatrix} \qquad J = \det \boldsymbol{F}$$

- Strain measures: Green-Lagrange strain

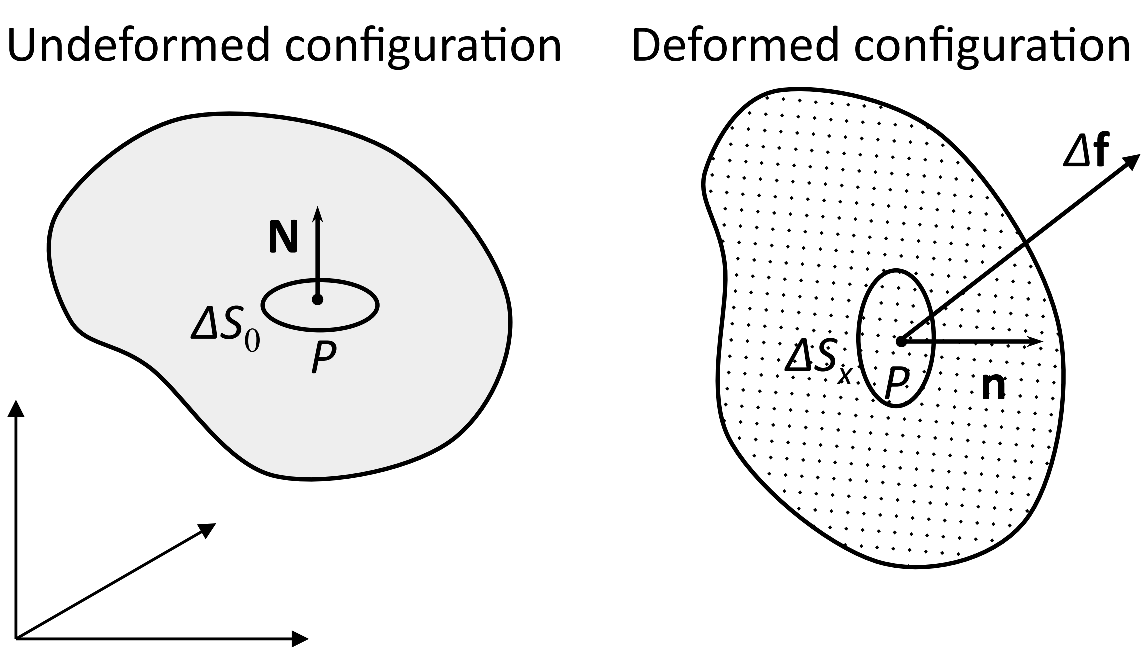
$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}) \qquad E_{ij} = \frac{1}{2}(F_{ik}^T F_{kj} - \delta_{ij})$$

- Stress measures:
  - The Cauchy stress (true stress) $\boldsymbol{\sigma} = J^{-1} \boldsymbol{P} \boldsymbol{F}^T$
  - The first Piola-Kirchhoff $\mathbf{P}(\mathbf{F})$
  - The second Piola-Kirchhoff $\boldsymbol{S} = \boldsymbol{F}^{-1} \boldsymbol{P}$

Stress and strain measures

- Cauchy stress (true stress): Force acts on the deformed configuration and the area in the deformed configuration.

- First PK stress (nominal stress): Force int he deformed configuration and the area in the undeformed configuration.



$$n \cdot \sigma dS = df = tdS$$

$$N \cdot \mathbf{P} dS_0 = df = t_0 dS_0$$

$$df = tdS = t_0 dS_0$$

For solid mechanics applications, it is instructive to directly develop the conservation equations in terms of the Lagrangian measures of stress and strain in the reference configuration (**P(F)** and **E**).

Small deformation vs. Large deformation

Small deformation hypothesis

- Small displacements: we equate the initial and deformed configurations, and set the equilibrium equation as the initial configuration (no update along time)

$$\boldsymbol{x} \simeq \boldsymbol{X}$$

- Small displacement gradient:

$$|\frac{\partial u_i}{\partial X_j}| \ll 1$$

- Under the small deformation hypothesis we can linearise the problem:

$$E_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\Big) \simeq \frac{1}{2}\Big(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\Big)$$

In this presentation we will assume Large deformations

Polar decomposition

Polar decomposition theorem for large deformations

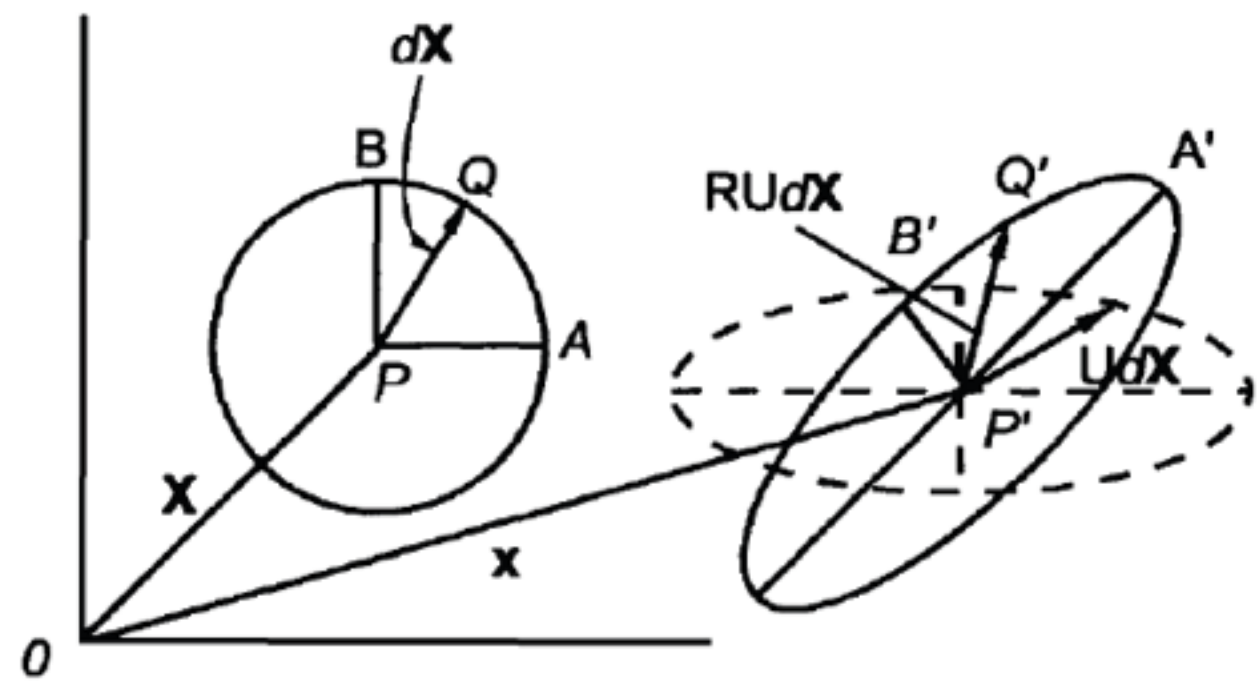
- Any gradient tensor **F** with a nonzero determinant can be multiplicatively decomposed into the product of an orthogonal matrix **R** and a positive definite symmetric tensor **U**

$$\mathbf{F} = \mathbf{R} \cdot \mathbf{U} \qquad F_{ij} = \frac{\partial x_i}{\partial X_j} = R_{ik} U_{kj}$$

Physical interpretation

- The material element d**X** first undergoes a pure stretching **U** and then rotation **R**

$$d\mathbf{x} = \mathbf{R} \cdot \mathbf{U} \cdot d\mathbf{X}$$

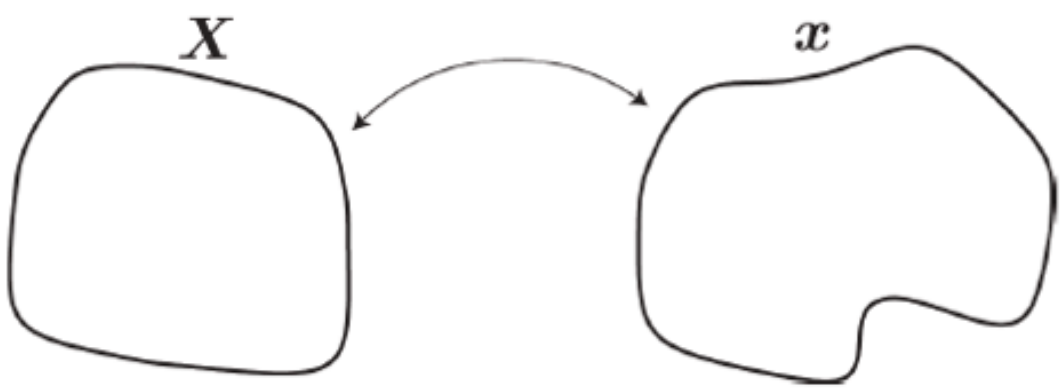


Conservation equations

Conservation of linear momentum with the first PK $\mathbf{P}(\mathbf{F})$.

$$\rho_0 \frac{\partial \mathbf{v}(\boldsymbol{X}, t)}{\partial t} = \nabla \cdot \mathbf{P} + \rho_0 \boldsymbol{b}$$

$$\frac{d}{dt} \int_V \rho_0 \mathbf{v} dV = \int_V \rho_0 \mathbf{b} dV + \int_{\partial V} \mathbf{t} dS$$



This must be complemented with the constitutive equation, which characterises the material behaviour:

$$S = S(E)$$

*The pair **P** and **F** is not especially useful for constructing constitutive equations, since **F** does not vanish in rigid body motion and **P** is not symmetric. Therefore constitutive equations are usually formulated in terms of the of the PK2 stress **S** and the Green strain **E**.*

Weak form: principle of virtual power

- Multiply the momentum equation by a test function w and integrate over the reference configuration

$$\rho_0 \frac{\partial \mathbf{v}(\boldsymbol{X},t)}{\partial t} = \nabla \cdot \mathbf{P} + \rho_0 \boldsymbol{b}$$

$$\int_{B_0} \nabla \cdot \boldsymbol{P} \cdot \boldsymbol{w} \, \mathrm{d}V + \int_{B_0} \boldsymbol{b}_0 \cdot \boldsymbol{w} \, \mathrm{d}V = \int_{B_0} \rho_0 \ddot{\boldsymbol{x}} \cdot \boldsymbol{w} \, \mathrm{d}V$$

- Apply integration by parts and the divergence theorem in the first integral

$$\int_{\partial B_0} \boldsymbol{P} \boldsymbol{n}_0 \cdot \boldsymbol{w} \, \mathrm{d}S + \int_{B_0} \boldsymbol{b}_0 \cdot \boldsymbol{w} \, \mathrm{d}V = \int_{B_0} \boldsymbol{P} \cdot \nabla \boldsymbol{w} \, \mathrm{d}V + \int_{B_0} \rho_0 \ddot{\boldsymbol{x}} \cdot \boldsymbol{w} \, \mathrm{d}V$$

- Since $\boldsymbol{w} = 0$ in the boundary and the applied traction $\bar{\boldsymbol{t}}_0 = \boldsymbol{P} \boldsymbol{n}_0$ at the traction boundary

$$\int_{\partial_t B_0} \bar{\boldsymbol{t}}_0 \cdot \boldsymbol{w} \, \mathrm{d}S + \int_{B_0} \boldsymbol{b}_0 \cdot \boldsymbol{w} \, \mathrm{d}V = \int_{B_0} \boldsymbol{P} \cdot \nabla \boldsymbol{w} \, \mathrm{d}V + \int_{B_0} \rho_0 \ddot{\boldsymbol{x}} \cdot \boldsymbol{w} \, \mathrm{d}V$$

Weak form: principle of virtual power

- It can also be deduced from the principle of virtual work

$$\boxed{\int_{\partial_t B_0} \bar{t}_0 \cdot w \, \mathrm{d}S + \int_{B_0} b_0 \cdot w \, \mathrm{d}V} = \boxed{\int_{B_0} P \cdot \nabla w \, \mathrm{d}V} + \boxed{\int_{B_0} \rho_0 \ddot{x} \cdot w \, \mathrm{d}V}$$

VW of boundary loads

VW of volumetric loads

VW of internal (elastic) forces

VW of inertial forces

VW of external loads

- Usually it is written as the equilibrium equation

$$W^{\mathrm{int}}(\mathrm{w}, \mathrm{x}) - W^{\mathrm{ext}}(\mathrm{w}, \mathrm{x}) + W^{\mathrm{kin}}(\mathrm{w}, \mathrm{x}) = 0$$

Discretization with FEM

- As standard, approximate unknown nodal position and test functions

$$\boldsymbol{x}_h^e(\boldsymbol{X}) = \sum_{a \in \Omega_0^e} N^a(\boldsymbol{\xi})\, \boldsymbol{x}^a \qquad \boldsymbol{w}_h^e(\boldsymbol{X}) = \sum_{a \in \Omega_0^e} N^a(\boldsymbol{\xi})\, \boldsymbol{w}^a$$

- And replacing in the weak form

$$\begin{aligned}\sum_e \int_{\Omega_0^e} \left(\rho_0 N^a N^b \,\mathrm{d}V\right) \ddot{\boldsymbol{x}}^a \cdot \boldsymbol{w}^b + \sum_e \int_{\Omega_0^e} \left(\boldsymbol{P}(N^a \boldsymbol{x}^a)\, \boldsymbol{\nabla}_0 N^b \,\mathrm{d}V\right) \cdot \boldsymbol{w}^b \\ = \sum_e \int_{\partial_t \Omega_0^e} \left(\bar{t}_0 N^b \,\mathrm{d}S\right) \cdot \boldsymbol{w}^b + \sum_e \int_{\Omega_0^e} \left(b_0 N^b \,\mathrm{d}V\right) \cdot \boldsymbol{w}^b,\end{aligned}$$

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{f}_{\text{int}}(\mathbf{x}) = \mathbf{f}_{\text{ext}}}$$

Solving the system of equations

- Static problems (no need to compute matrix M)
  - Solve the matrix equations (direct solver vs. iterative solver)

$$\mathrm{K}\mathrm{x} = \mathrm{f}$$

- Dynamic problems: the discretised equations are coupled with a set of ODEs
  - Select a time step Δt
  - Adopt and algorithm to advance in discrete time steps: Newmark scheme
  - Stability and accuracy need to be considered

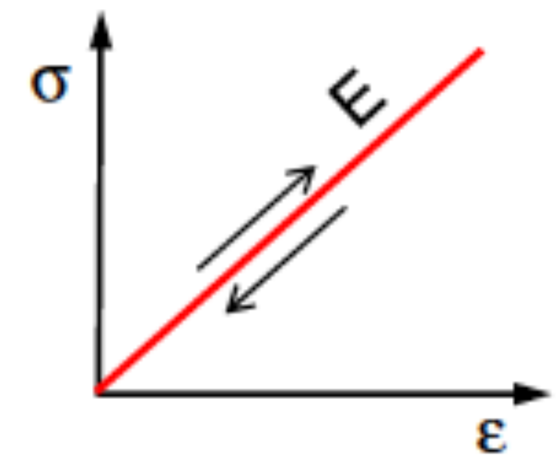
$$\mathrm{M}\ddot{\mathrm{x}} + \mathrm{K}\mathrm{x} = \mathrm{f}$$

Elasticity

Linear Elasticity

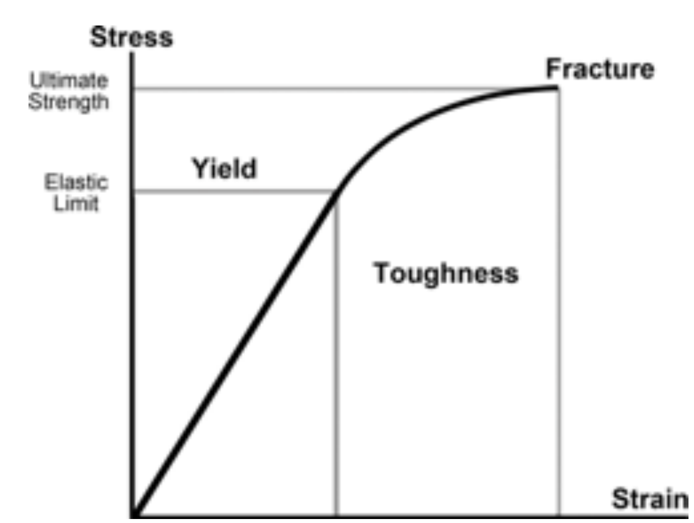
What does linear elastic mean?

- An elastic material wants to resist a change in shape. It returns to its original shape.
- Linear relationship between how hard you squish it and how much it deforms (stress and strain).



Do real materials that we care about actually have these properties?

- In general, solids like plastic and metal behave more or less as linear elastic materials unless you really bend or squish them.
- Above some threshold amount of force, materials become both nonlinear and non-elastic. After that point, materials will usually permanently bend and eventually crack.



Isotropic Elasticity

- Elasticity: one-to-one relation between stress and deformation.

- Isotropic elasticity: the behaviour is independent of the orientation. The elasticity tensor C does not depend on the basis.

$$\boxed{\boldsymbol{S} = \mathbb{C}\boldsymbol{E}} \qquad \boxed{\mathrm{S}_{ij} = \mathbb{C}_{ijkl}\mathrm{E}_{kl}}$$

$$C_{ijkl} = C_{ijlk}$$
$$C_{ijkl} = C_{jikl}$$
$$C_{ijkl} = C_{klij}$$

C is a 4th order tensor with 81 parameters but.. applying symmetries it reduces to 21 parameters

Recall: we use total Lagrangian formulation, where 2PK ***S*** and the G-L strain ***E*** are the natural choice. It's easy to recover 1PK ***P*** from:

$$\mathbf{F} \cdot \mathbf{S} = \mathbf{P}^T$$

Isotropic elasticity

In the case of isotropic elasticity, the elasticity tensor C is constant and solely characterised by a pair of elastic moduli, e.g., two Lame's constants λ and μ

$$\mathbb{C} = \lambda \mathbf{I} \otimes \mathbf{I} + 2\mu \mathbf{I}$$

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right)$$

Lamé equations

$$\mathbf{S} = \lambda tr(\mathbf{E})\mathbf{I} + 2\mu \mathbf{E} \qquad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

Usually the following parameters are used

$$G = \mu$$

Shear modulus

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$

Young modulus

$$\nu = \frac{\lambda}{2(\lambda + \mu)}$$

Poisson

$$\mathbb{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

Young modulus and poisson

Poisson effect: when a material is compressed in one direction, it usually tends to expand in the other two perpendicular directions:.

The limits of the Poisson ratio are:

- $\nu=0$: no influence of lateral stresses to the strain, no lateral contraction (cork)
- $\nu=0.5$: incompressible material, means there is no volume change under loads (rubber)
- $\nu=0.2\text{-}0.3$: typical values for linear elastic material (ceramics and metals)

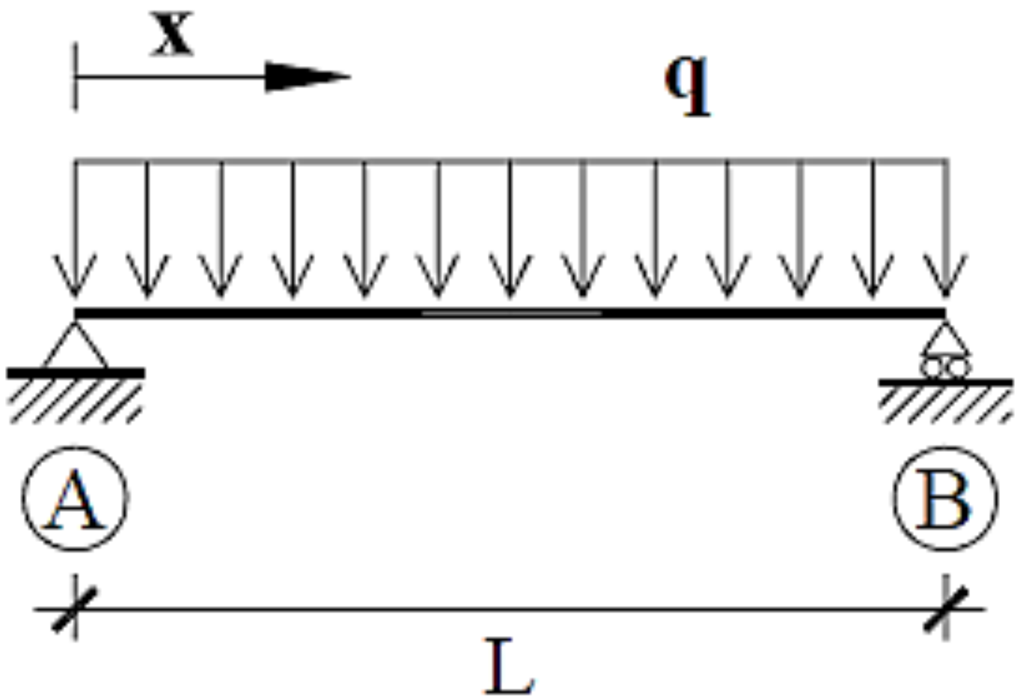
Young's modulus, which is also known as the elastic modulus, is a mechanical property of linear elastic solid materials. It defines the relationship between stress and strain in a material.

Material	Youngs Modulus /GPa
Mild Steel	210
Copper	120
Bone	18
Plastic	2
Rubber	0.02

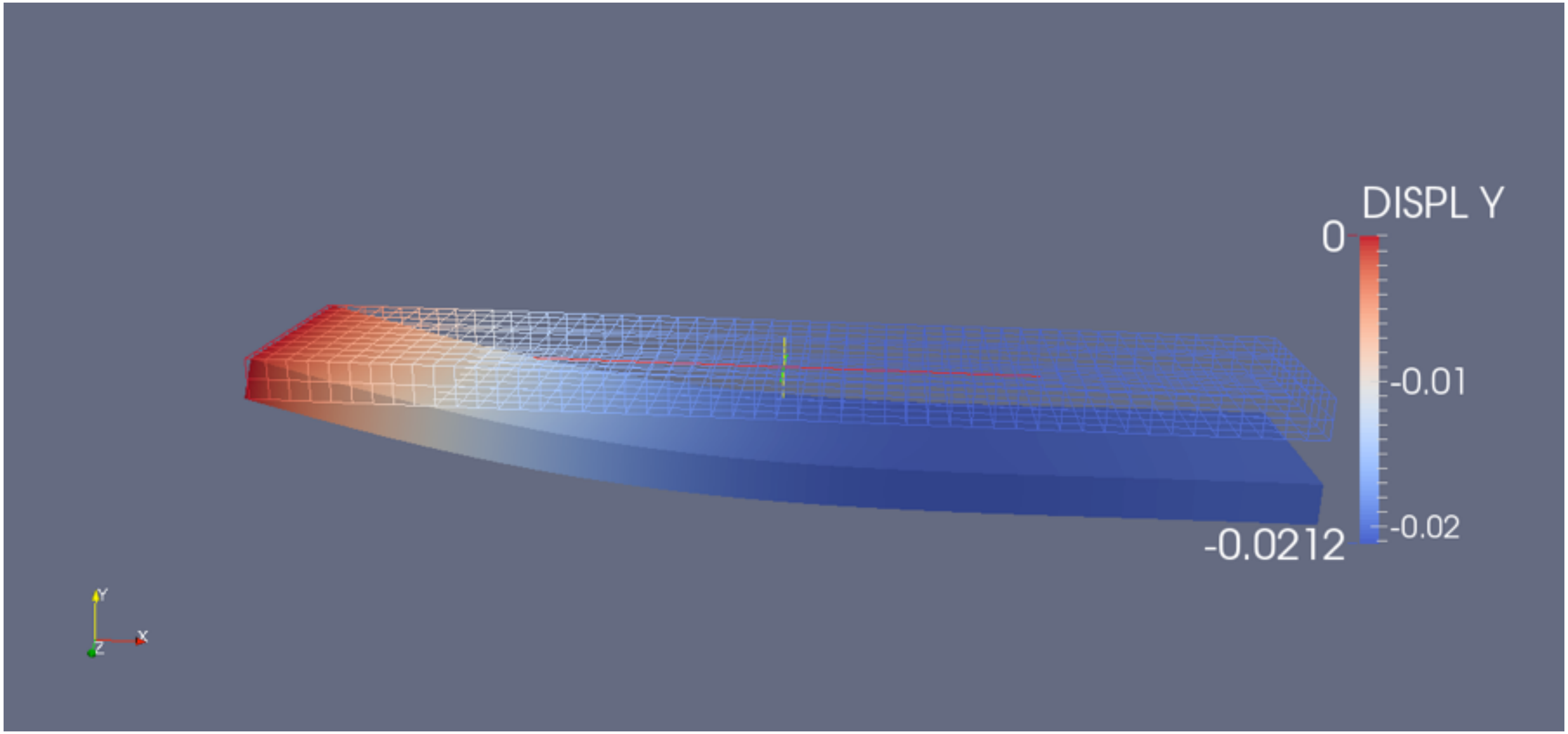
Simply supported beam

Supported bridges and high way road.

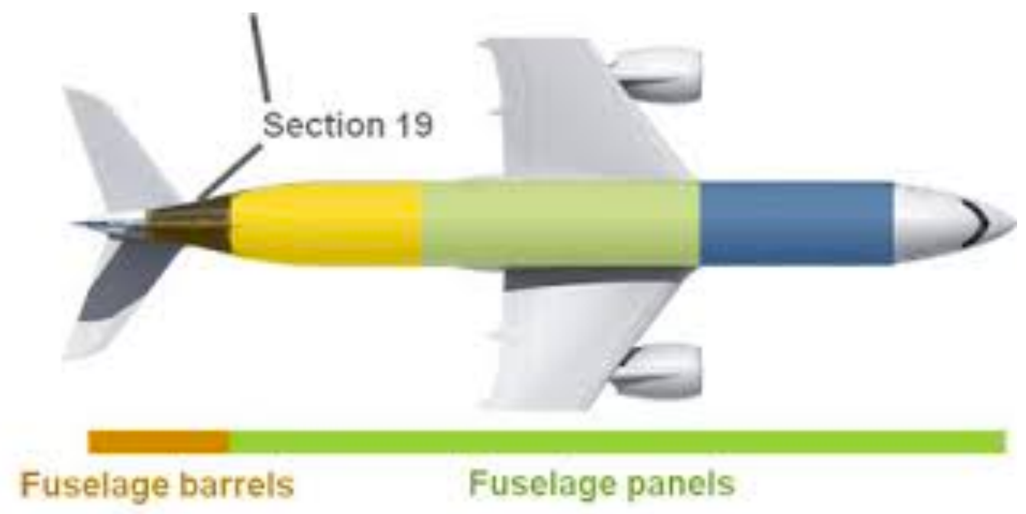
- E = 2.1e+11
- v = 0.3
- dimensions:
  - Real: 100 x 50 x 2
  - Model: 50 x 50 x 2 (symmetry BC)
- pressure = 0.5

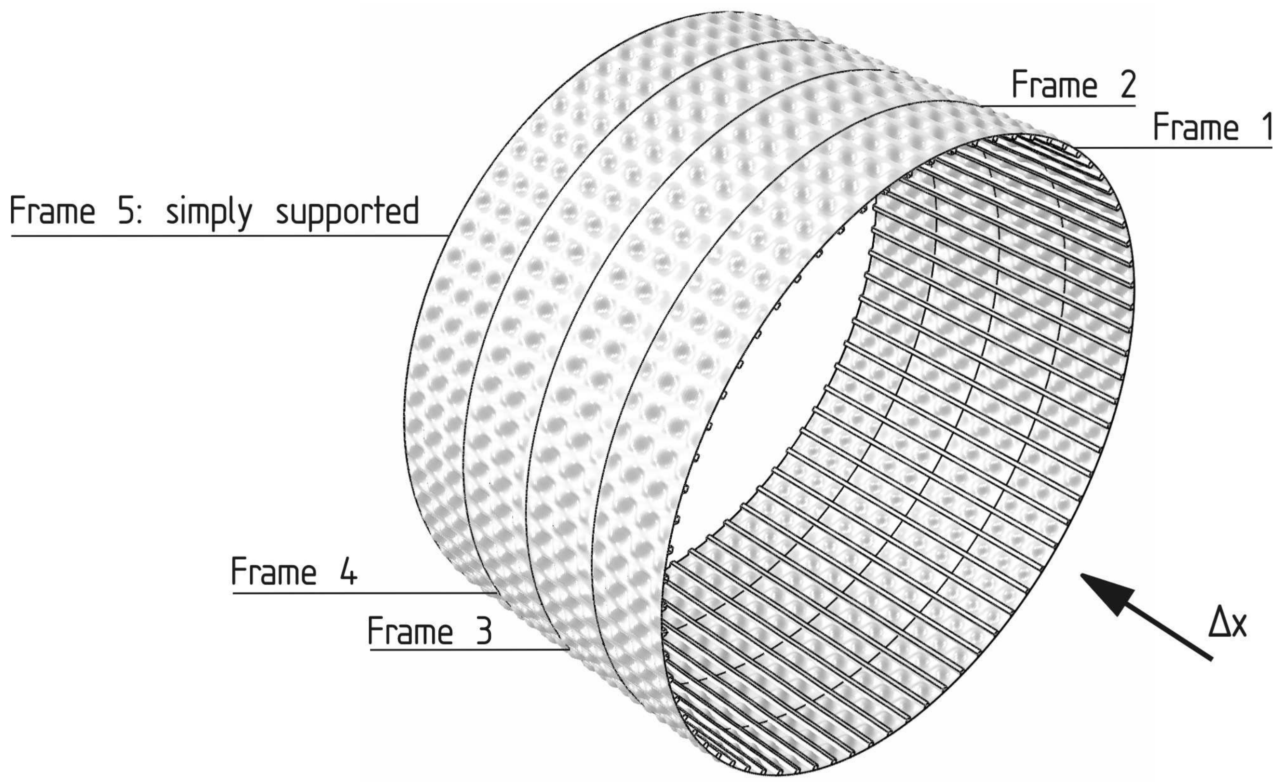


Simply supported beam



Full barrel fuselage



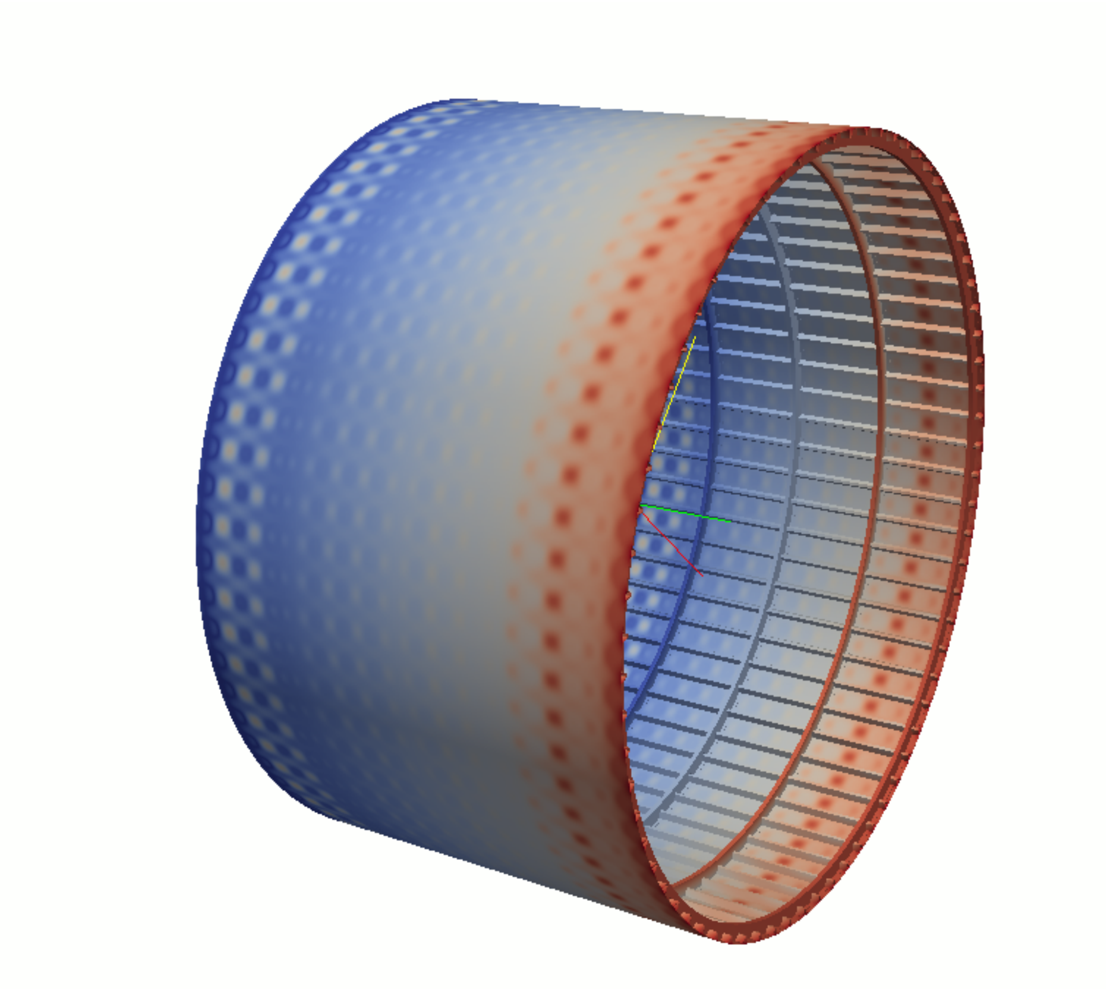


E = 70000 , ν=0.3

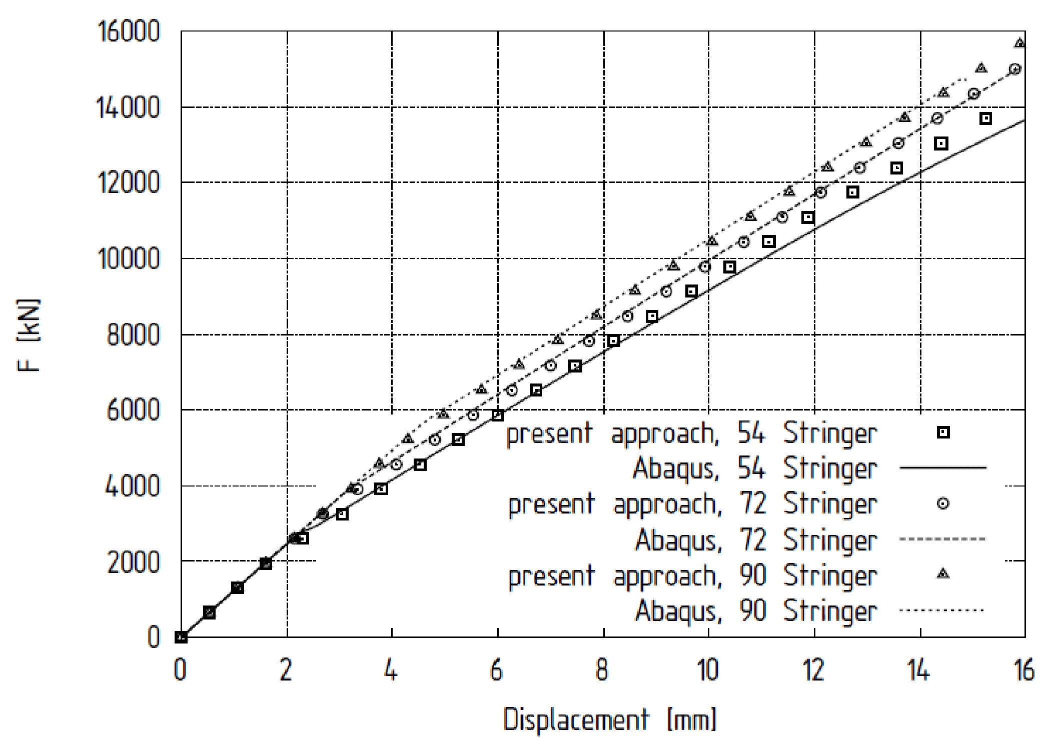
Isotropic material behaviour

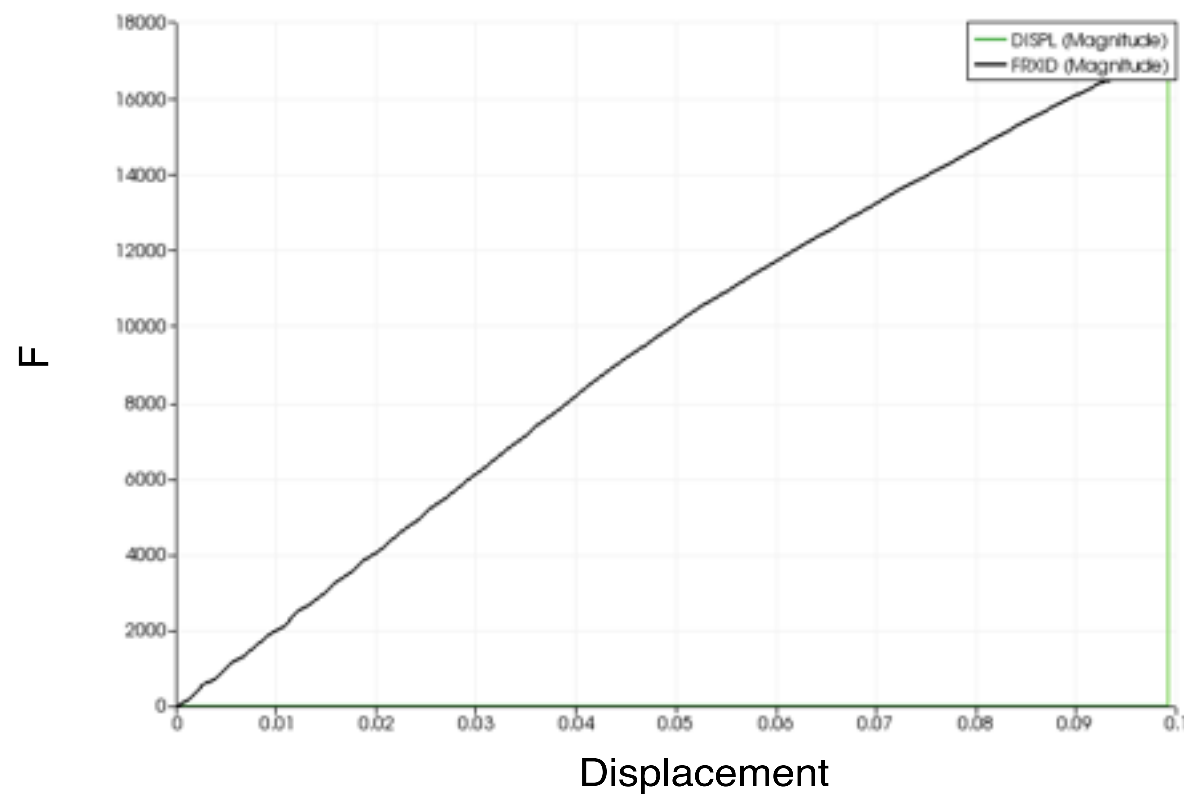
Unixaial compression loading

Full barrel fuselage: postbuckling behaviour



Buckling is a mathematical instability, leading to a failure mode. It is caused by a bifurcation in the solution to the equations of static equilibrium.





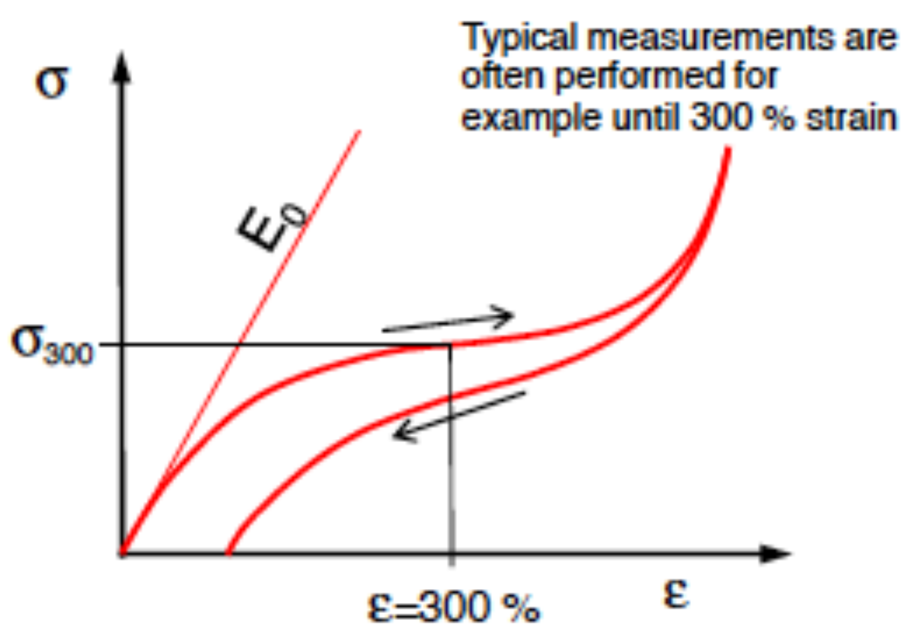
Hyperelasticity

Hyperelasticity

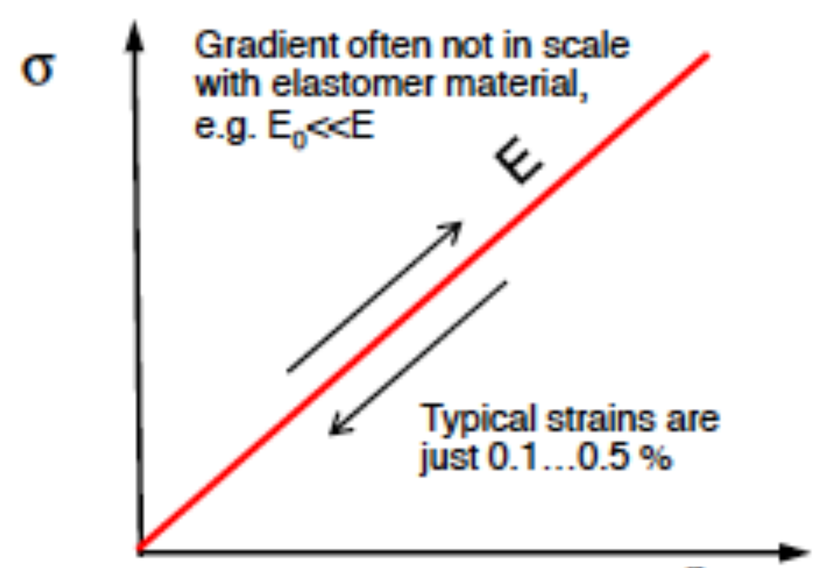
Definition and properties

- Materials that exhibit elastic behaviour through large strains to 300%.
- Stress is a function of current total strain (independent of history).
- Stress-strain relationship derives from a strain energy density function (elastic potential) $\mathrm{W(E)}$ instead of a constant factor , such that

$$\mathbf{S} = \frac{\partial W(\mathbf{E})}{\partial \mathbf{E}}$$



Elastomer material behavior



Linear elastic material
(like brittle steel or ceramic)

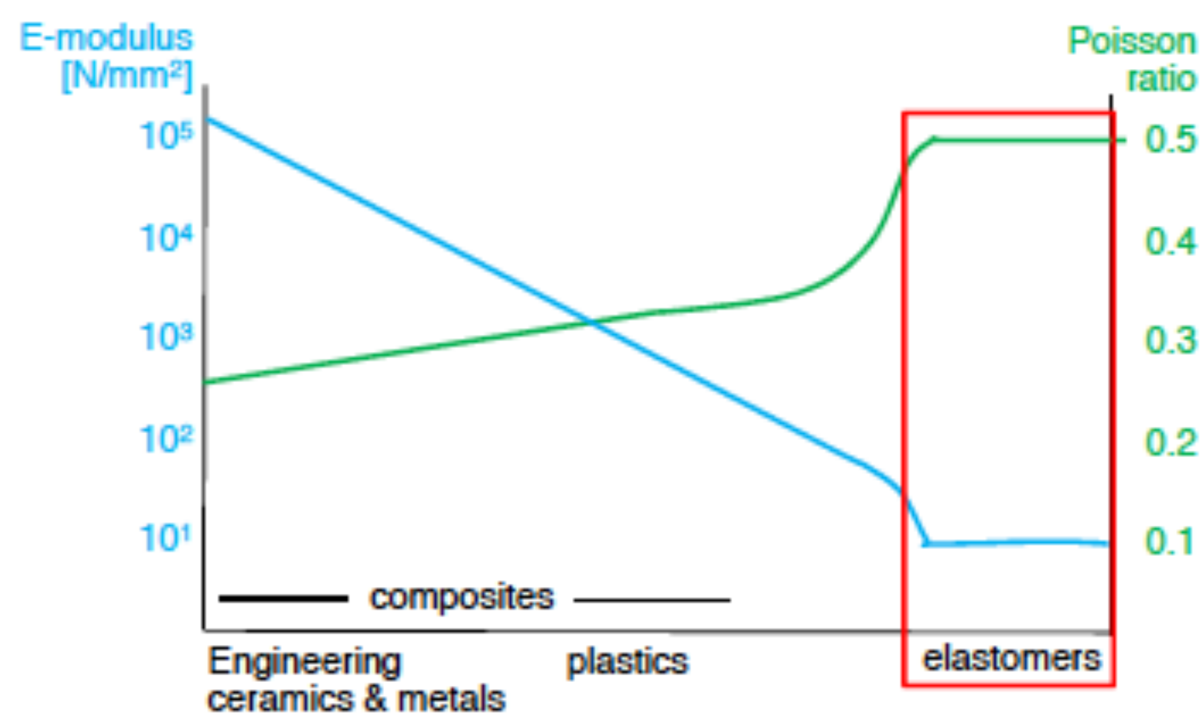
Hyperelasticity

Generally comes with incompressibility (J=1)

- The volume preserves during large deformation.
- Mixed formulation: completely incompressible hyperelasticity (pressure p is additional unknown).
- Penalty formulation: nearly incompressible hyperelasticity (linearization).

Typical materials

- Rubber material
- Plastic - particularly foams
- Biological tissues



Constitutive models in hyperelasticity

- The description of the strain energy density W is much more complex, compared to linear elastic material, where S is a linear function of E.
- In general W is a function of the stretch invariants $W=f(I_1,I_2,I_3)$.

$$W=\sum_{i+j=1}^{N} C_{ij}(I_1-3)^i(I_2-3)^j+\sum_{k=1}^{N}\frac{1}{D_k}(J-1)^{2k}$$

Classical hyper elastic models

- Neo-hookean $$W=C_{10}(I_1-3)+\frac{K}{2}(J-1)^2$$
- Mooney-Rivlin $$W=C_{10}(I_1-3)+C_{01}(I_2-3)+\frac{K}{2}(J-1)^2$$
- Holzapfel $$W=\frac{a}{2b}e^{b(I_1-3)}-\frac{a}{2}(I_1-3)+\frac{a_f}{2b_f}\left\{e^{b_f(I_4-1)^2}-1\right\}+\frac{K}{2}(J-1)^2$$

What is the right model to describe my material?

- If the strain is below apron. 5-10%, for many applications the simple Hooke's law is accurate enough to describe hyperelastic materials, so the time consuming nonlinear analysis can be replaced by a very quick linear one.

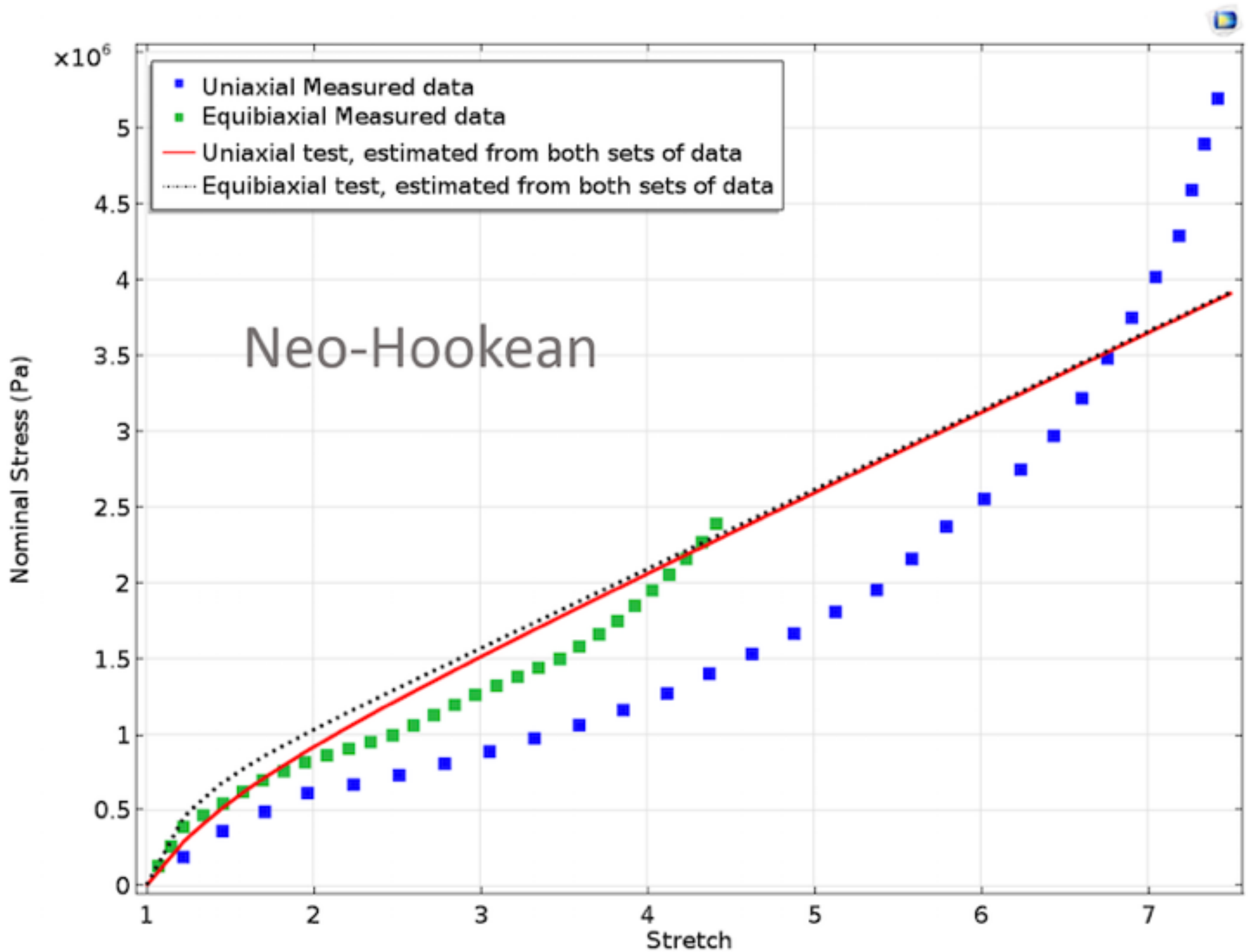
- If the strain becomes bigger but we don't have many test data, it is a good idea to start as a rough estimate with the most simple model, Neo-hookean, where the two necessary material contacts can be obtained from the initial shear and initial bulk modulus.

$$C_{10} = \frac{G}{2} \qquad K = \frac{E}{3(1-2\nu)}$$

K is the bulk modulus and represents the ratio between uniform stress and uniform strain (~inverse of compressibility)

- If more tests are available, we select the best suitable model…

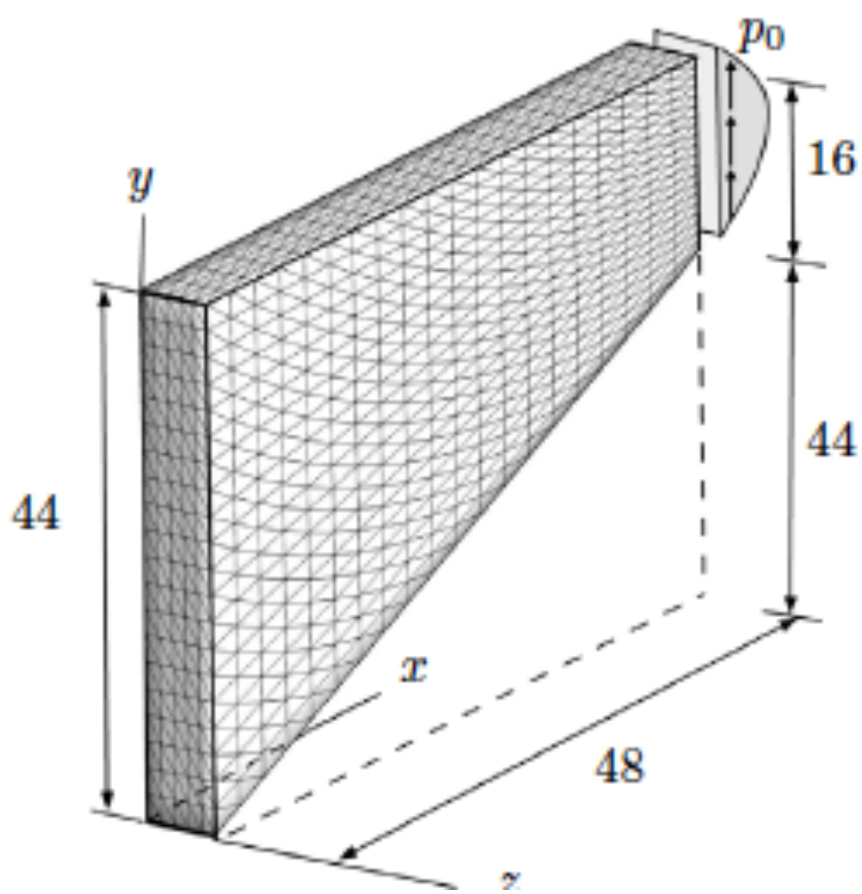
Neo-hookean material: uniaxial test

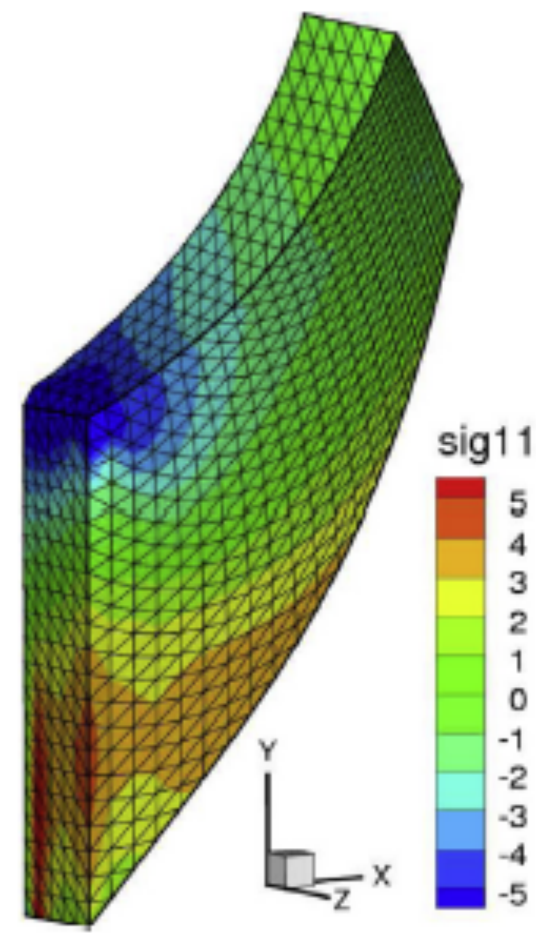


Fitting Measured Data to Different Hyperelastic Material Models
Chandan Kumar | June 24, 2015 (COMSOL)

Neo-hookean material: Cook's membrane test

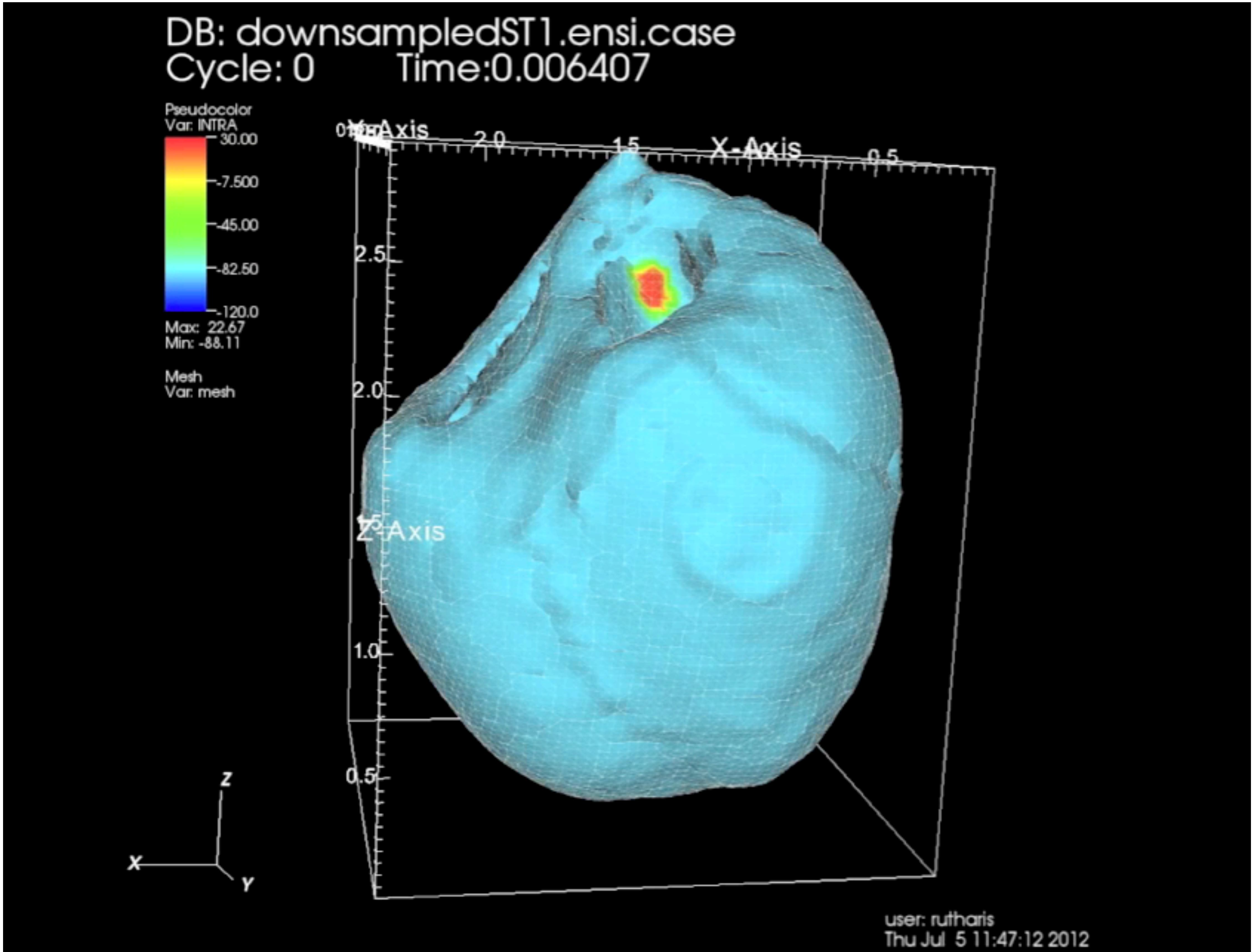
Soft material (~biological tissue): E = 200 ν=0.35





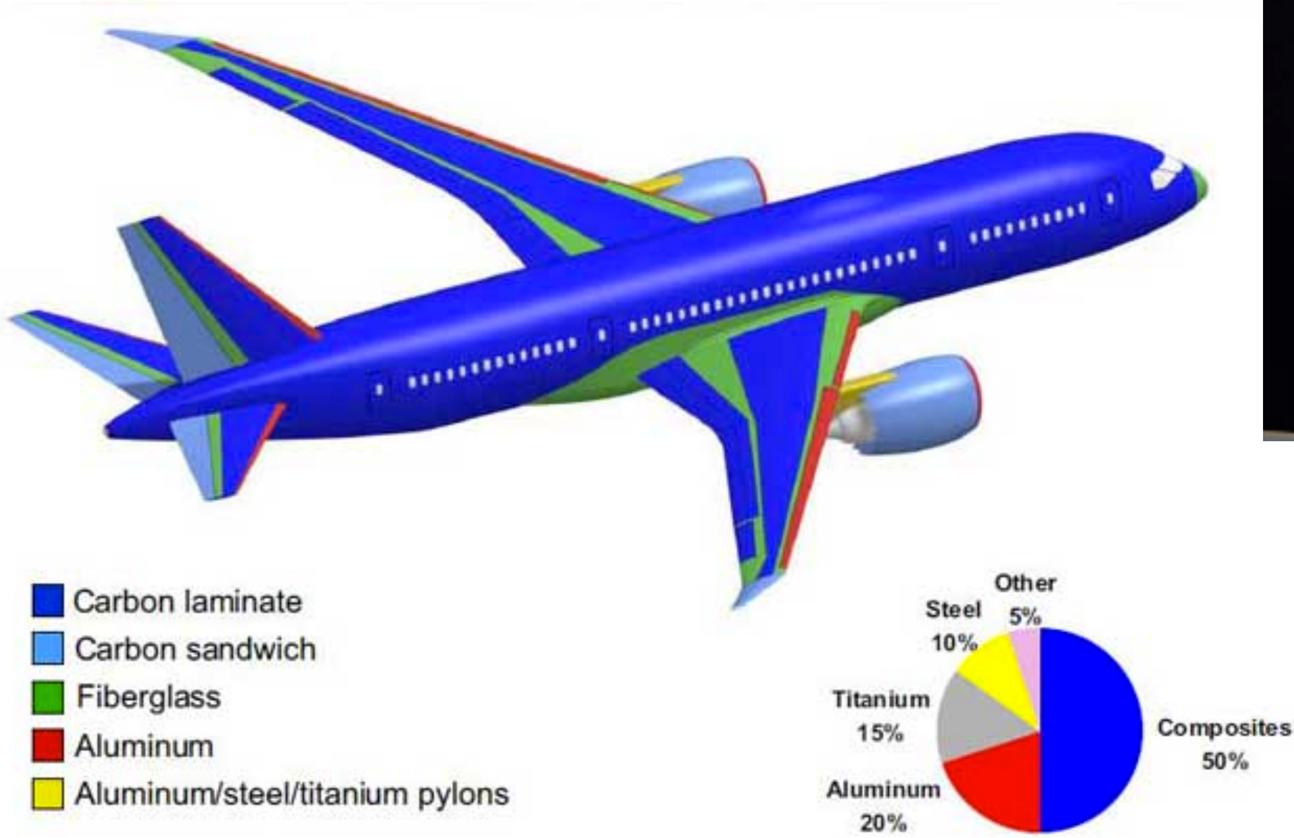
Robust numerical calculation of tangent moduli at finite strains based on complex-step derivative approximation and its application to localization analysis
Masato Tanaka, Masaki Fujikawa, Daniel Balzani, Jörg Schröder

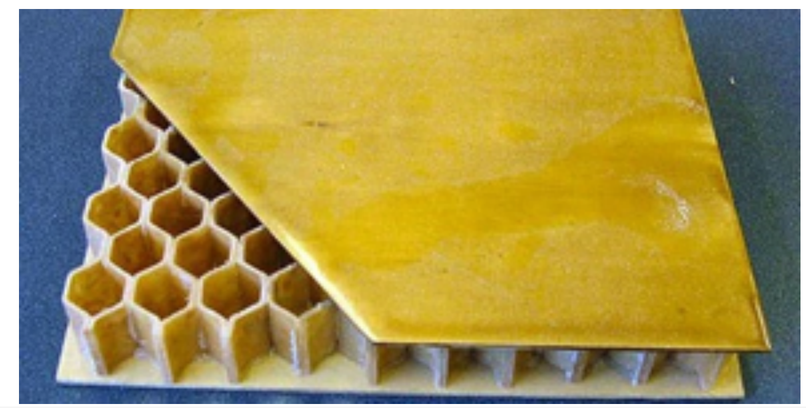
HGO material: Cardiac model

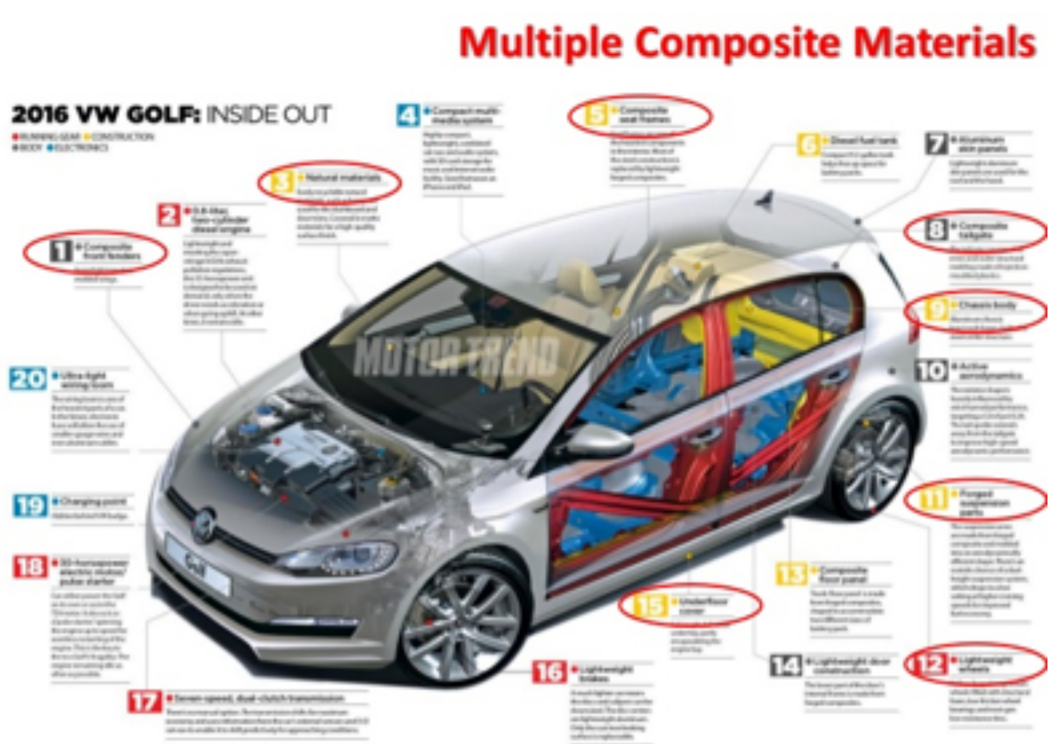


Composite materials

Composite materials







Composite materials

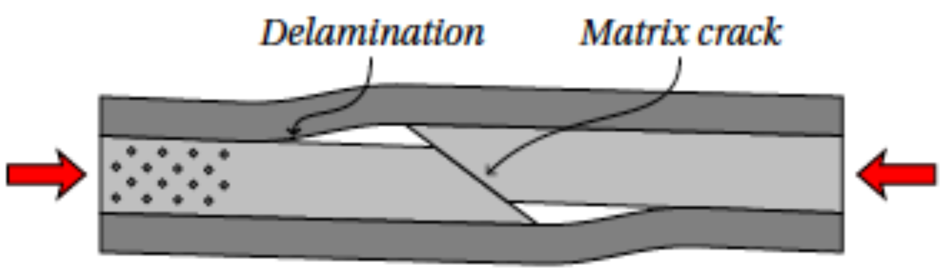
Why composite laminates?

- High strength/stiffness and light weight
- Excellent Thermal/Erosion Resistance
- Anisotropic Mechanical/Thermal Behavior
- Design optimization

The deficiencies in the models representing mechanical behaviour of composites have tremendous effects on the costs of composite structures

How to improve constitutive models?

- Simulate different failure scenarios where the analytical are no longer valid



Composite materials

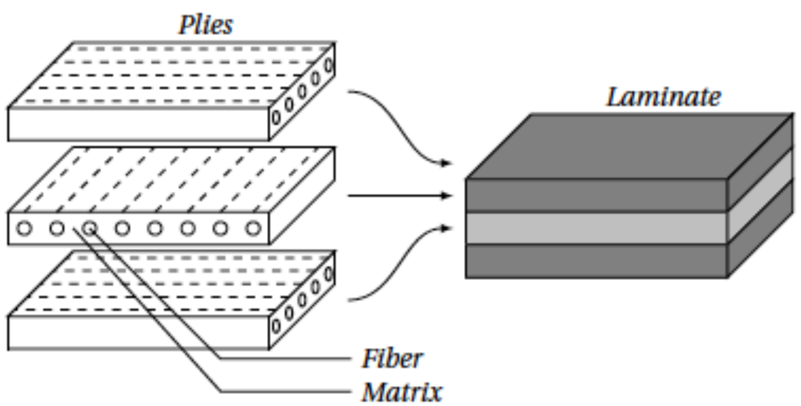
Fiber + **Matrix** = **Composite**

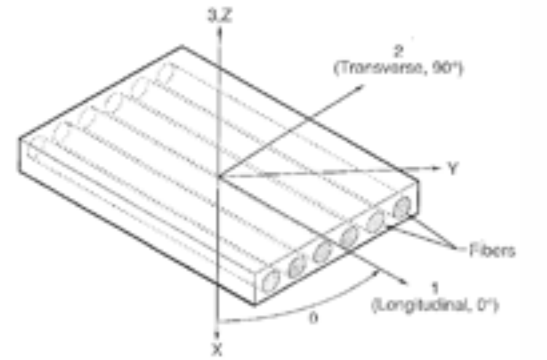
Fiber
- Glass
- SiC
- Aramid/ Kevlar
- Carbon

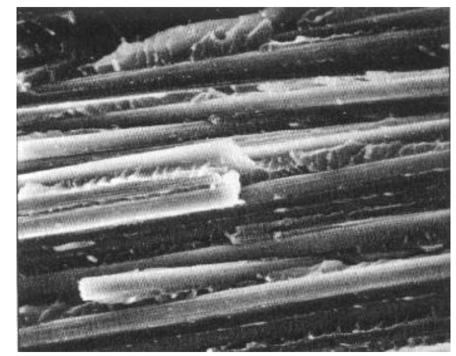
Matrix
- Epoxy/Plastic
- Ceramic
- Metal

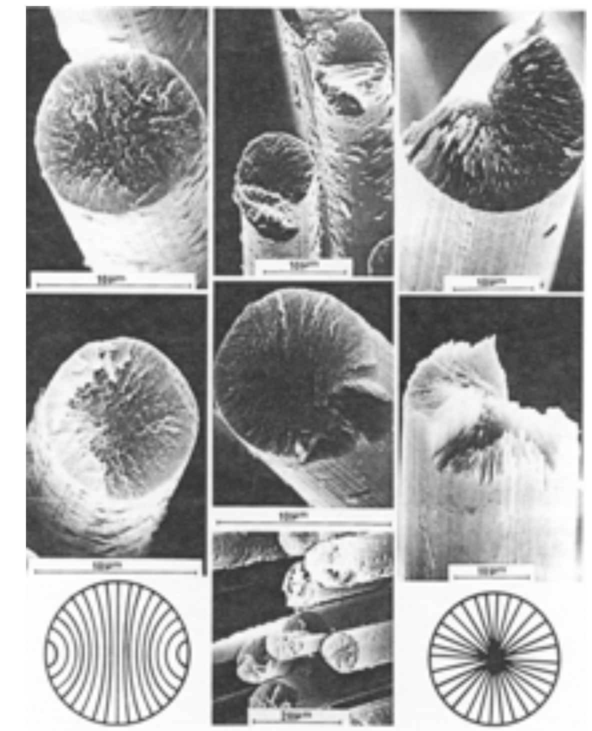
Composite

Fiber reinforced plastic composite
- GFRP
- CFRP
  - Laminate
  - Woven









- The composite is a system which consists of fibres in a resin or similar medium (usually called the matrix).
- The important strength and stiffness characteristic are provided by the high strength fibres.
- The fibres are usually shown as a schematic, in practice they will be very small diameter and scattered though the matrix in a ply.
- It is important to consider both the fibres and the matrix in the material stiffness and strength considerations.

Composite materials: damage and failure

Damage models

- In elastic materials, the mechanical response of a material is described by the relation

$$\sigma(\varepsilon)$$

- For more involved materials, such as composites, that may develop damage, we need to define a constitutive model with internal variables

$$\sigma(\varepsilon, D)$$

- The simplest damage model is the scalar damage model:

$$\sigma = (1 - d)\mathbb{C}\varepsilon \qquad \sigma_{ij} = (1 - d)C_{ijkl}\varepsilon_{kl}$$

The internal variables track the material state. The determination of a reliable set of internal variables is a critical point in any constitutive model.

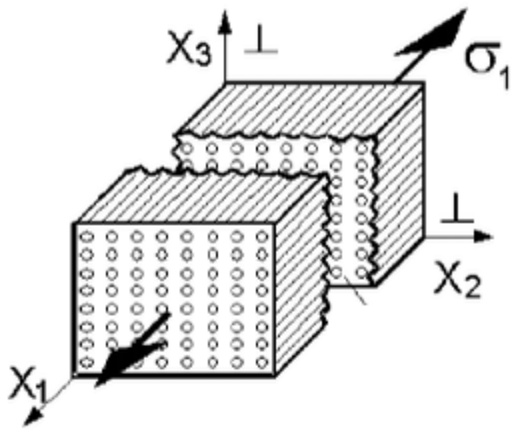
Typical internal variables are: temperature, kinematics, state variables.

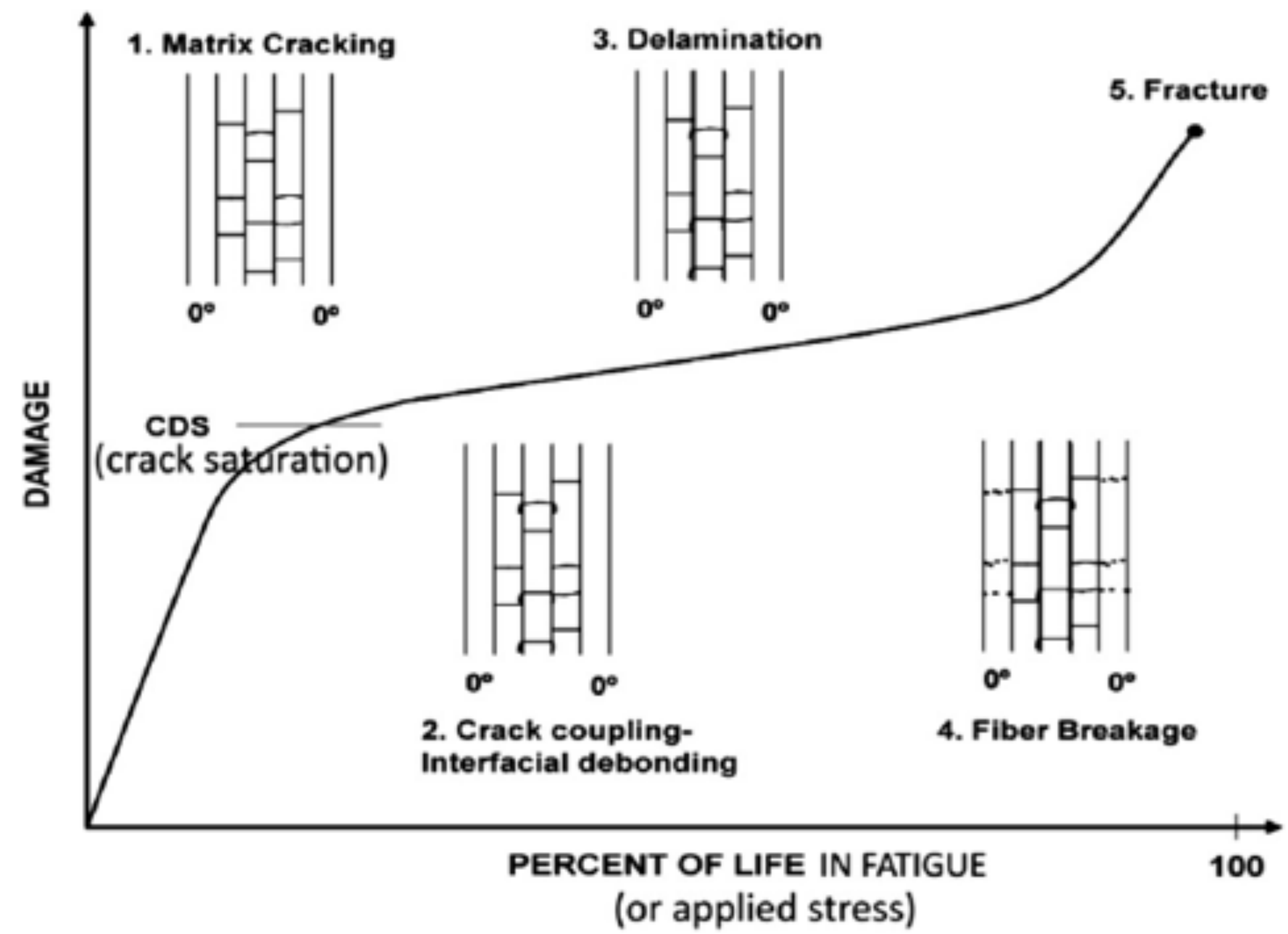
- Fro a damage model it is necessary to define the damage onset and the damage evolution (damage laws).

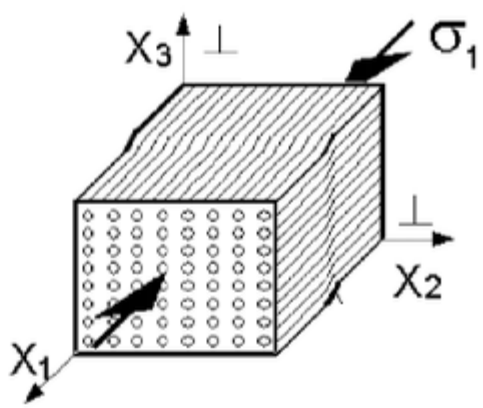
Damage models

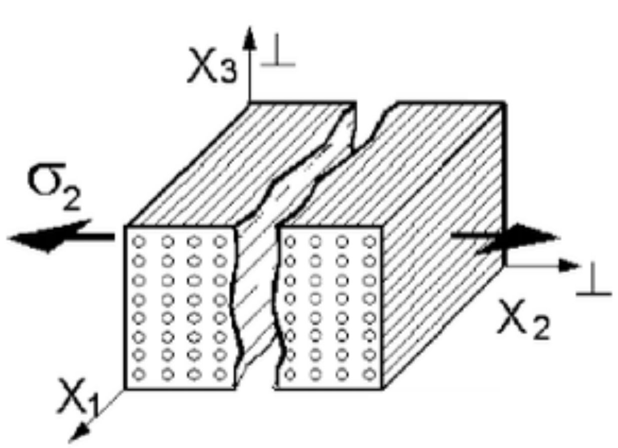
Failure process

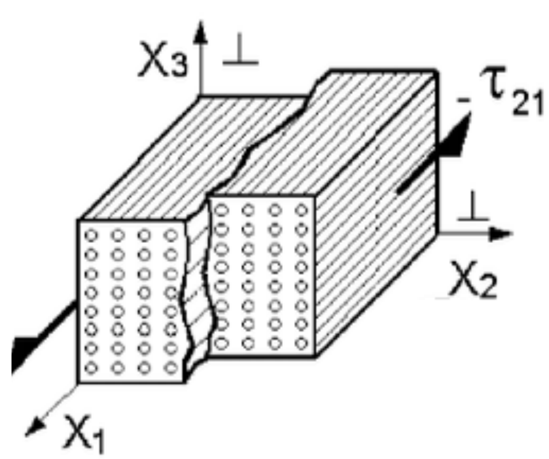
Source: Jamison RD et al.: Characterization and analysis of damage mechanisms in tension–tension fatigue of graphite/epoxy laminates. Effects of Defects in Composite Materials. ASTM STP 836; 1984. p. 21–55.

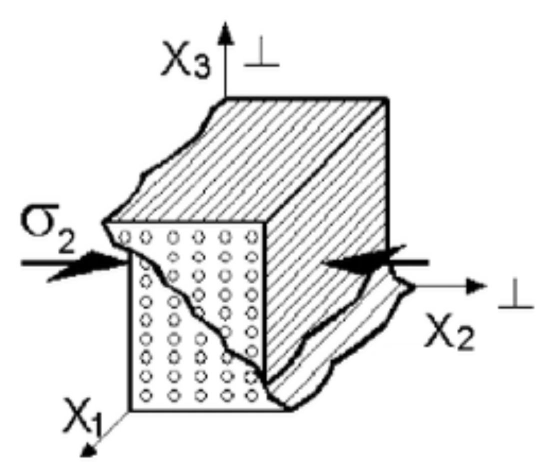




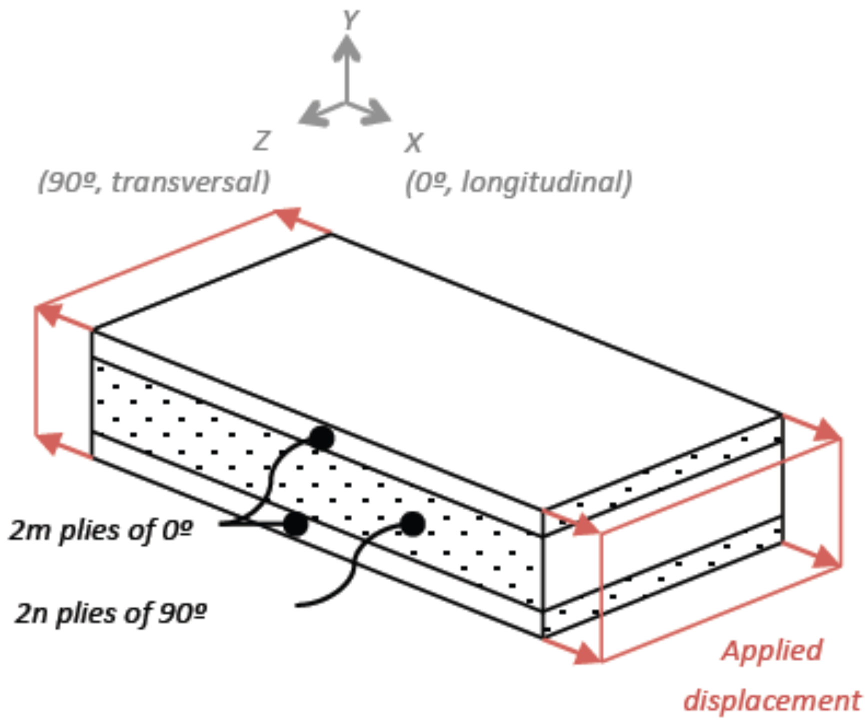


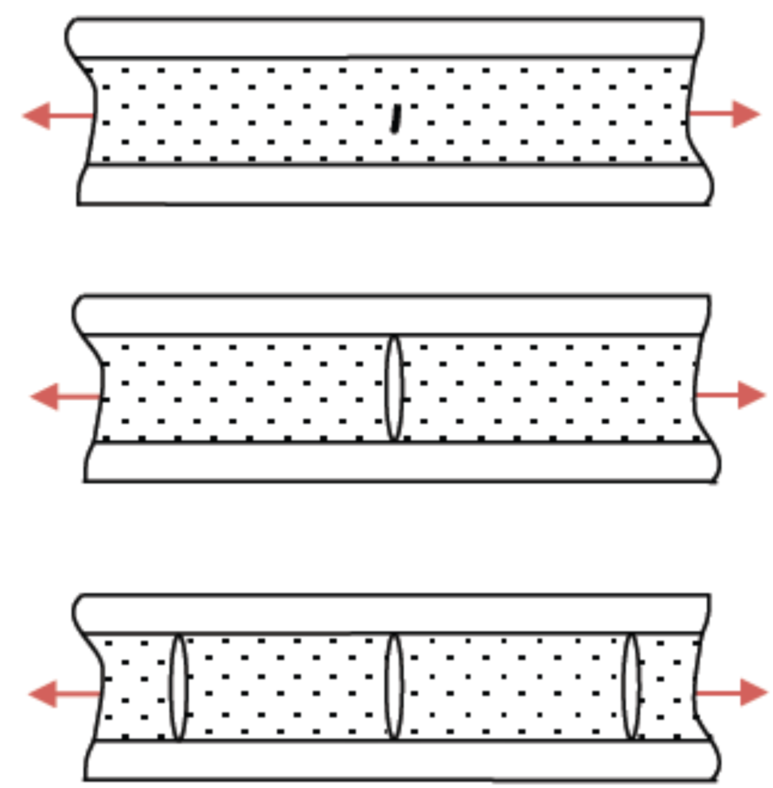


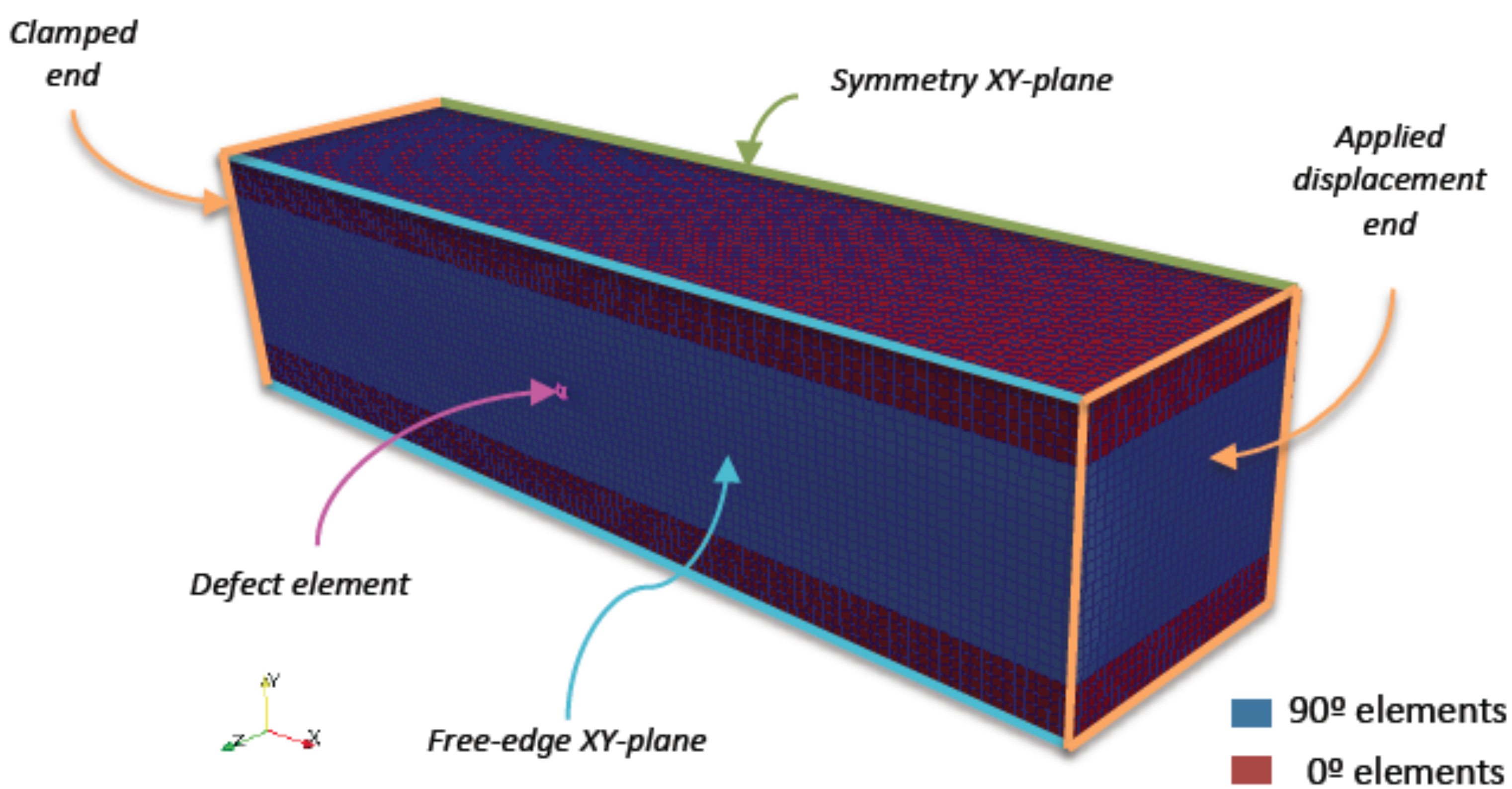




Matrix cracking

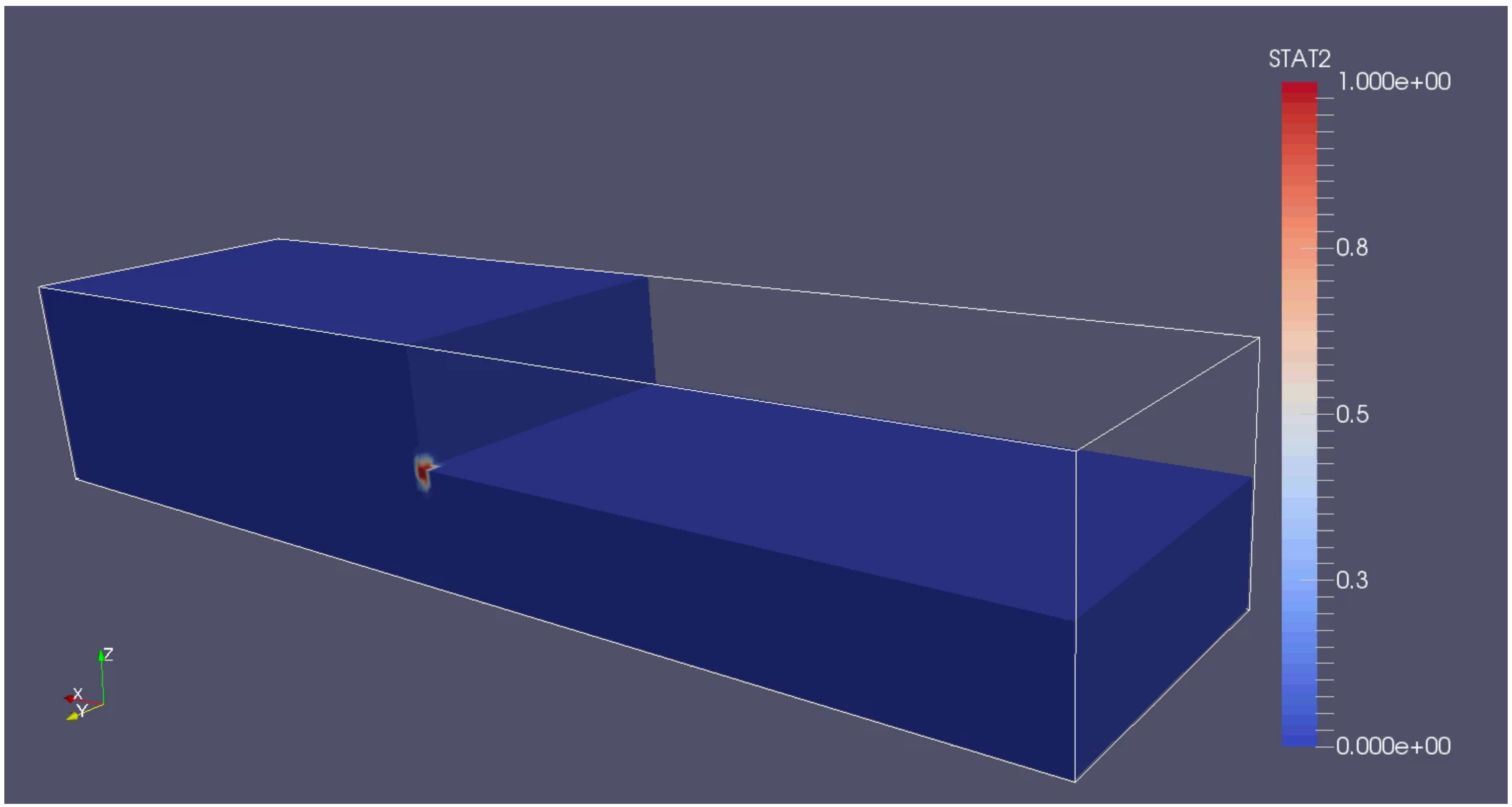






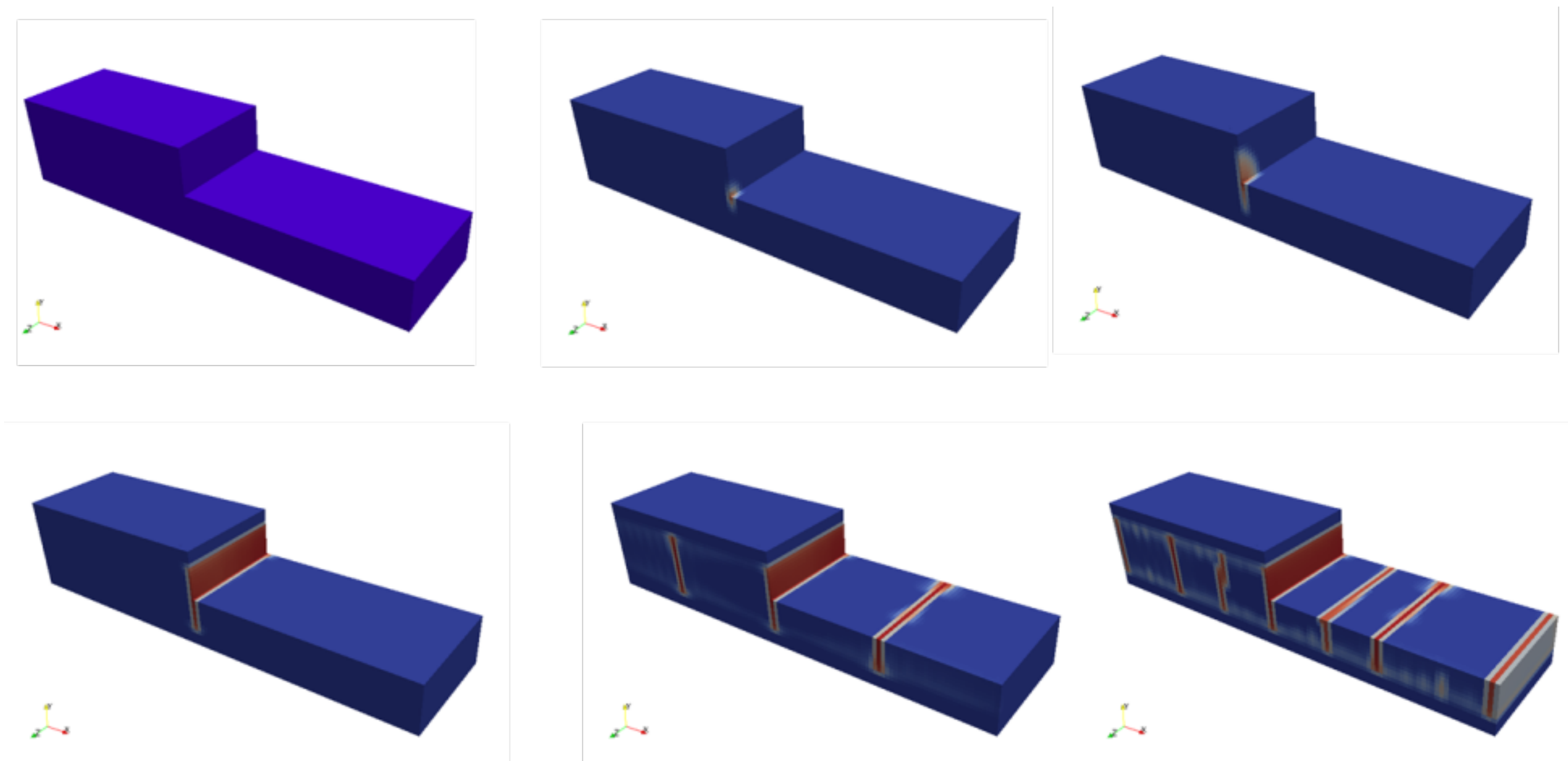
Matrix cracking

Transverse cracking scheme: onset and accumulation up to the final failure



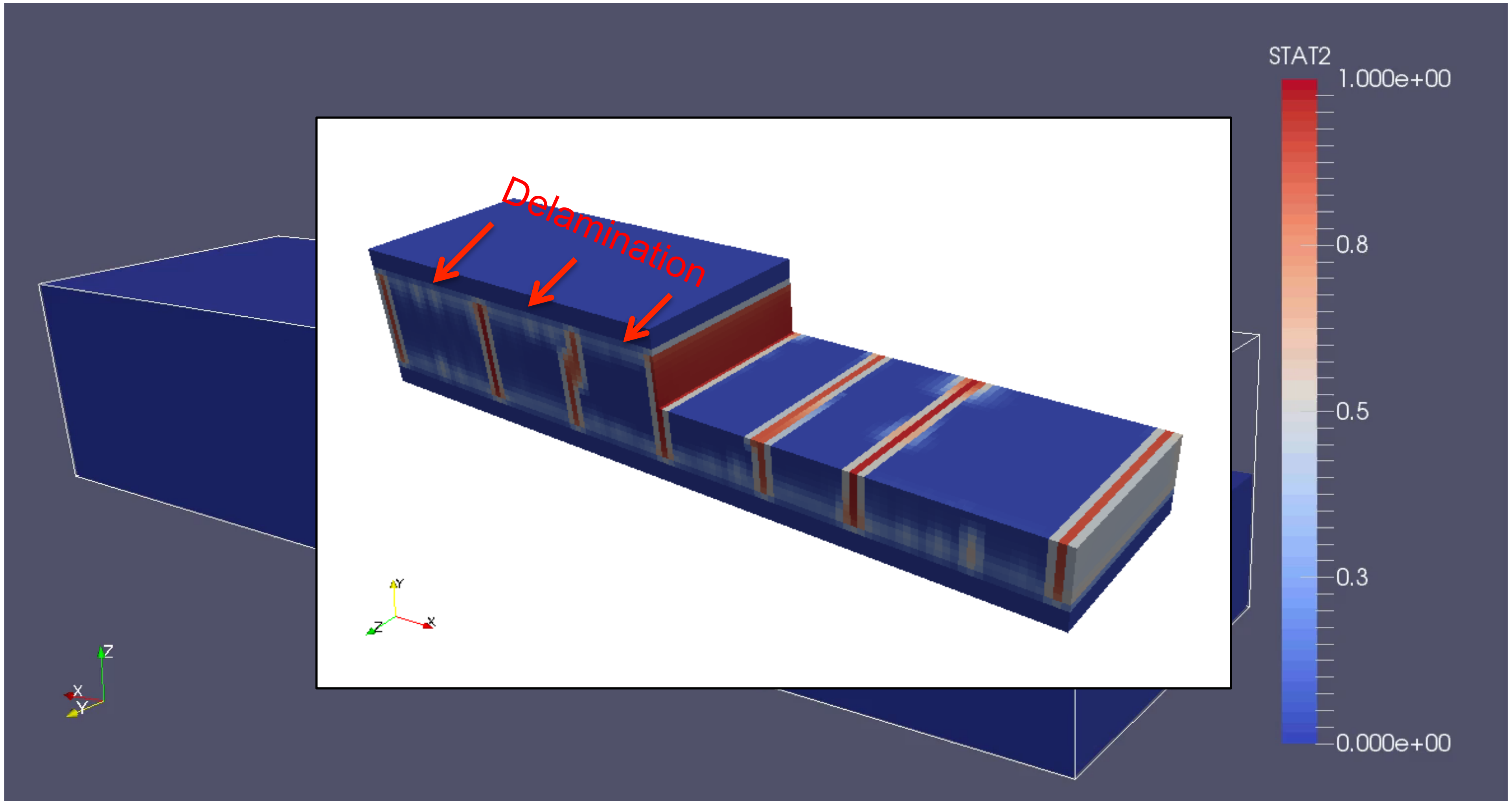
Matrix cracking

Transverse cracking scheme: onset and accumulation up to the final failure



Matrix cracking

Transverse cracking scheme: onset and accumulation up to the final failure



Matrix cracking

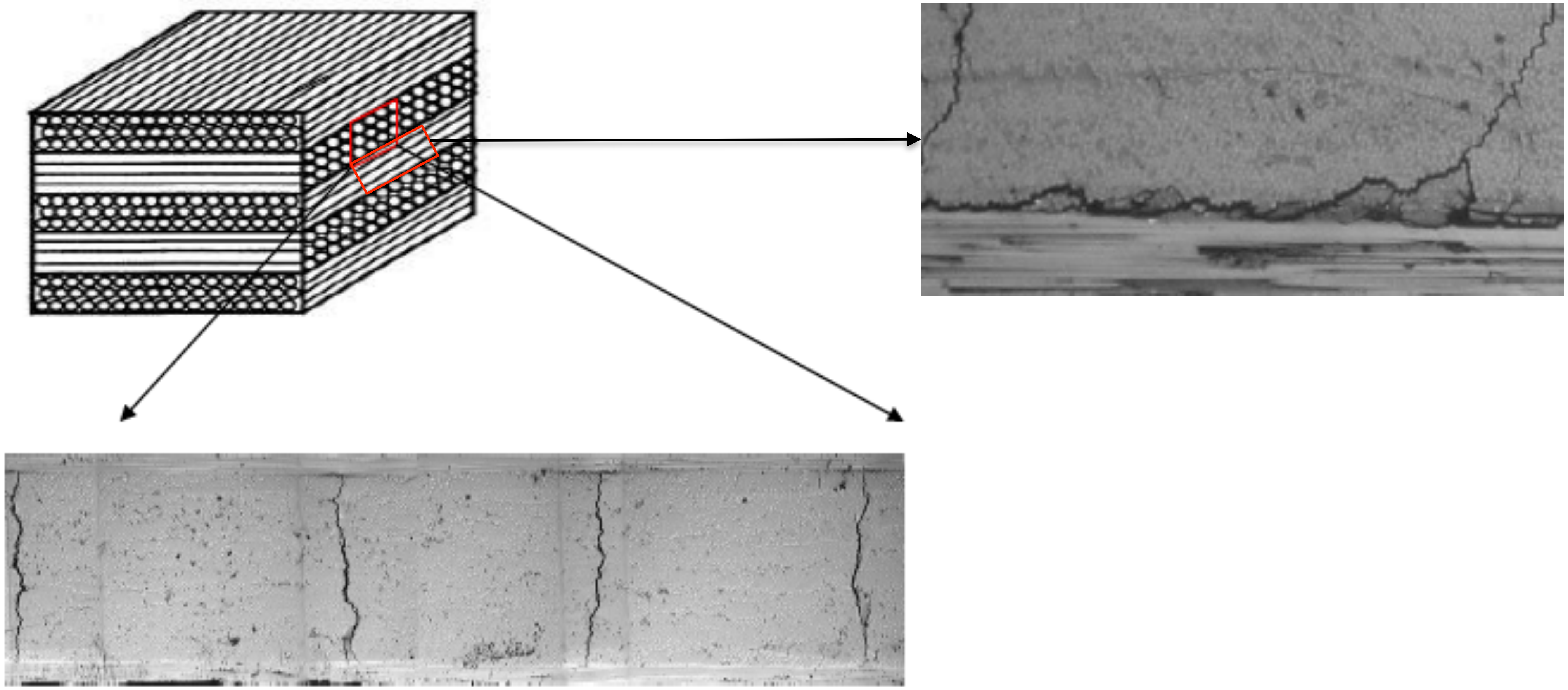


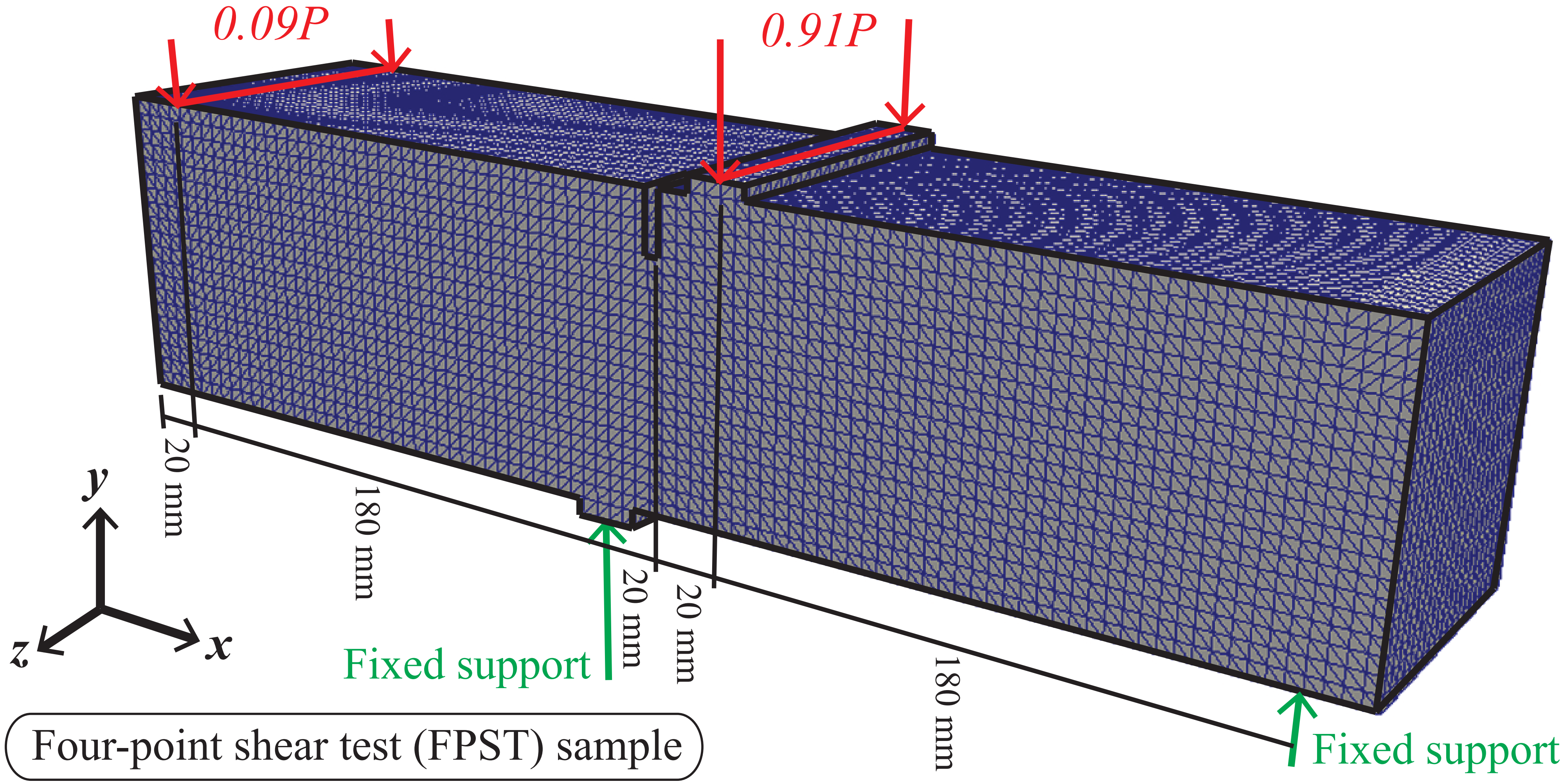
Figure 3. Cross-ply laminate with transverse matrix cracks in 90-layer.

[Bailey, J.E and Parvizi, A. (1981). On fiber debonding Effects and the Mechanisms of Transverse-Ply Failure in Cross-Ply Laminates ofGlass/Fiber/Themroset Composites, J. mat. Sci., 16: 649-659]

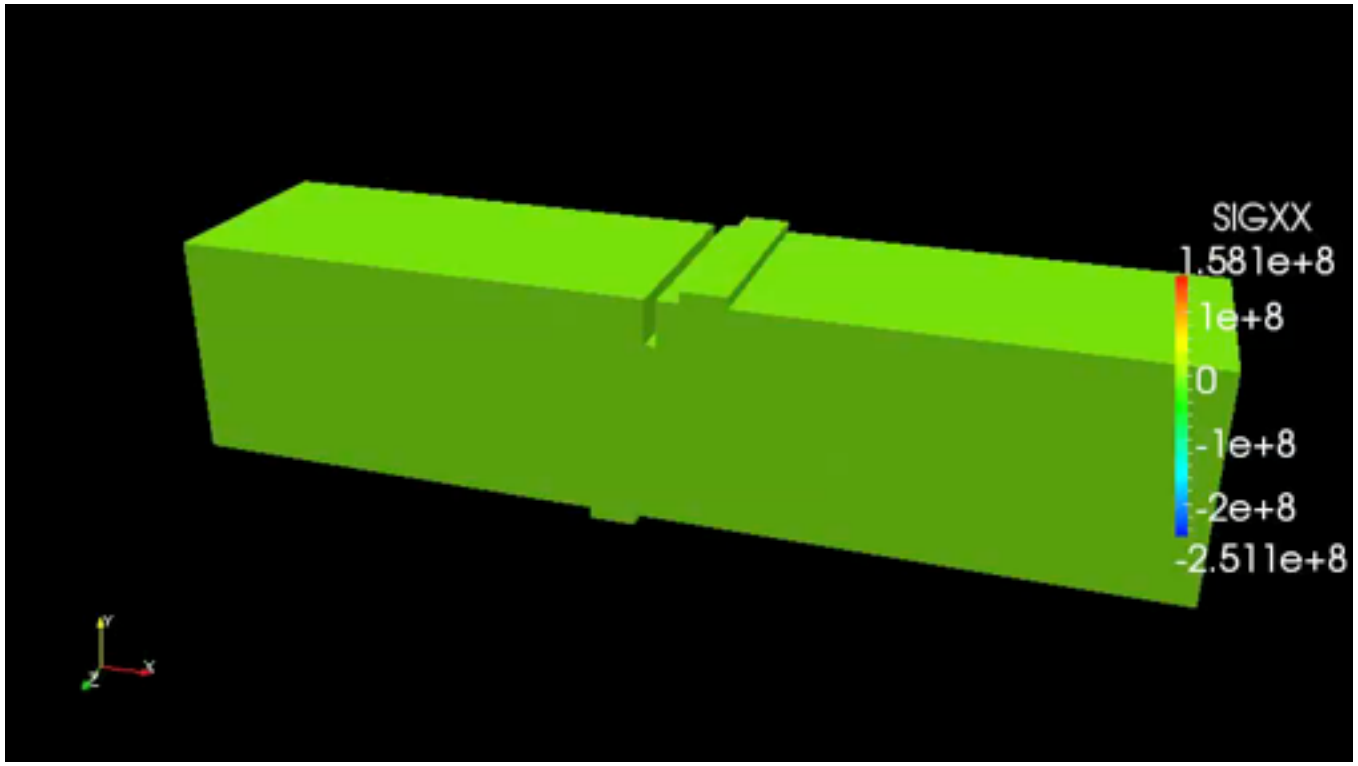
Fracture

Linear isotropic material (concrete)

$$
\begin{aligned}
E &= 200MPa \\
\nu &= 0.3
\end{aligned}
$$

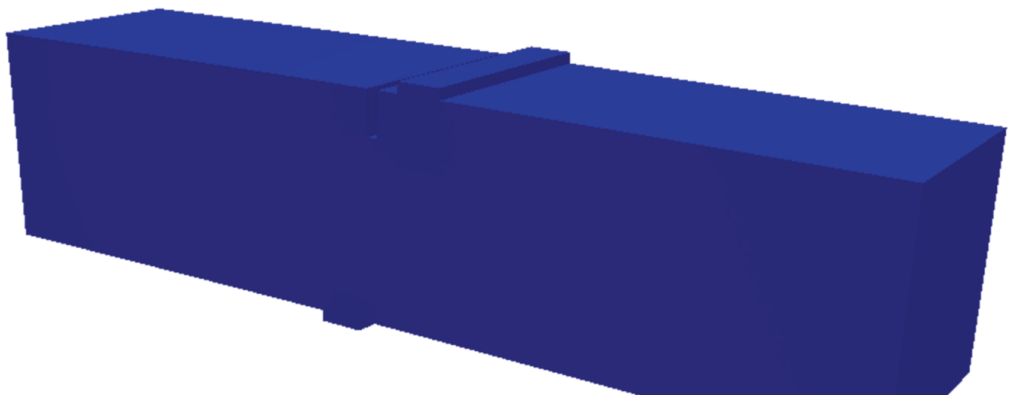


Four points shear test

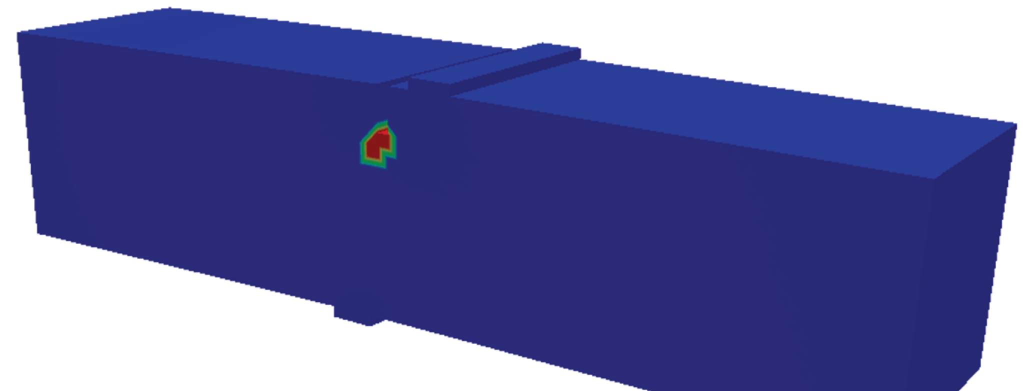


Four points shear test

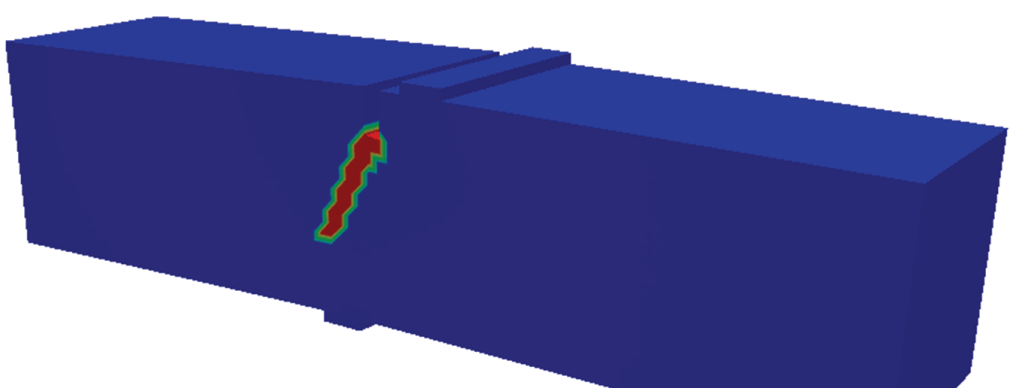
Crack evolution



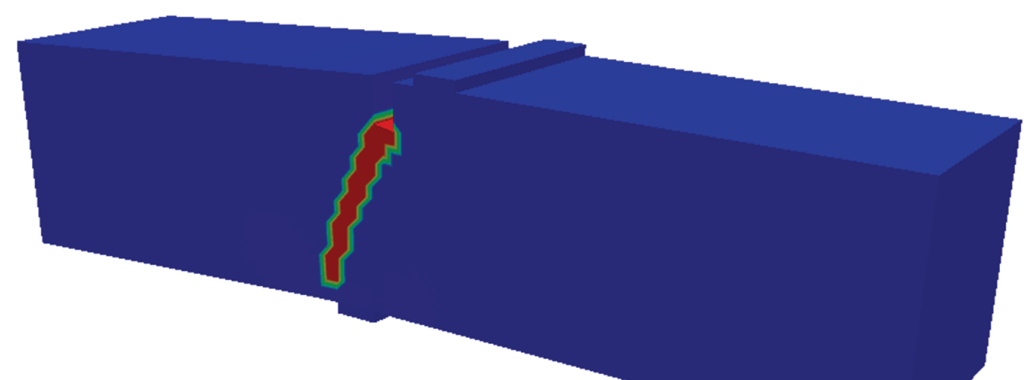
Step 1. *Crack initiation*



Step 2. *Maximum load*



Step 3. *Crack propagation*

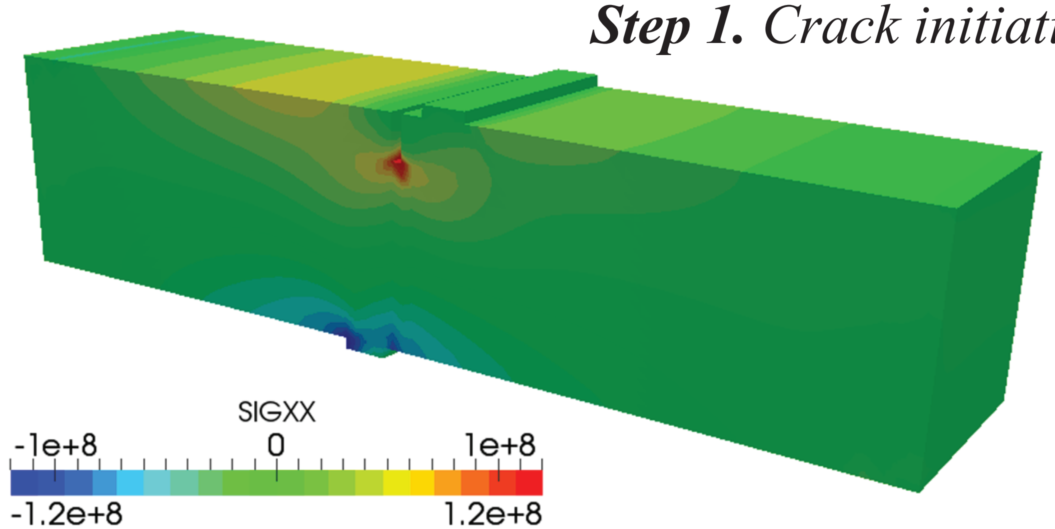


Step 4. *Final loading*

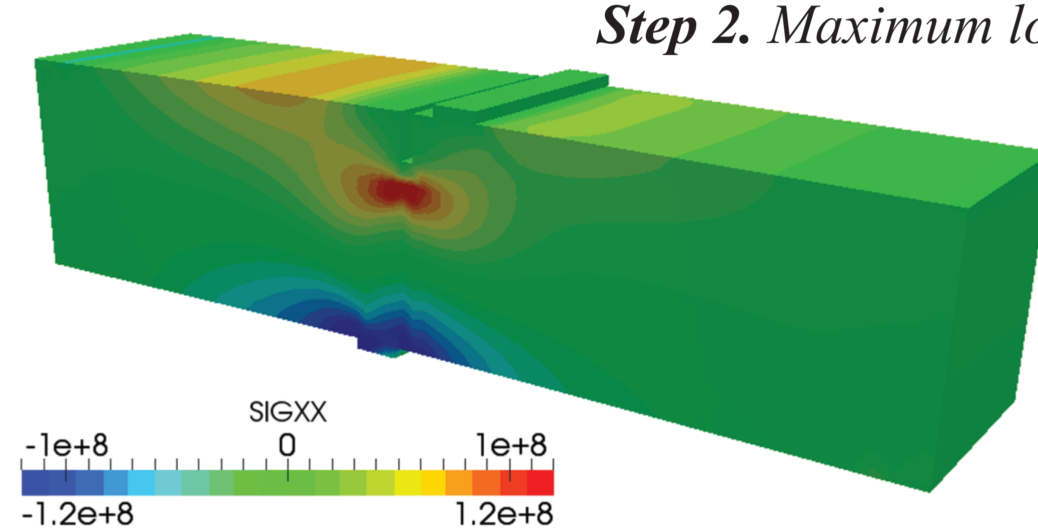
Four points shear test

Stress evolution

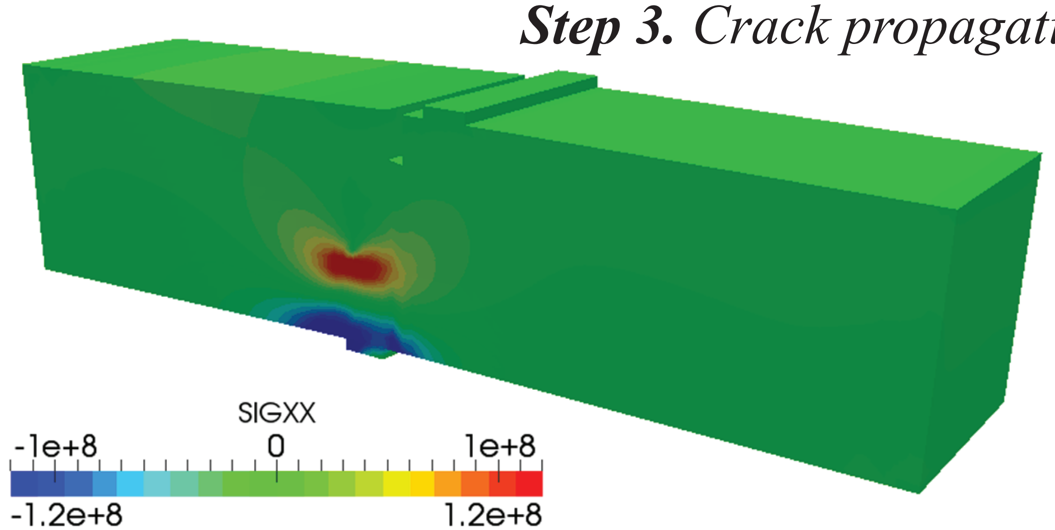
Step 1. *Crack initiation*



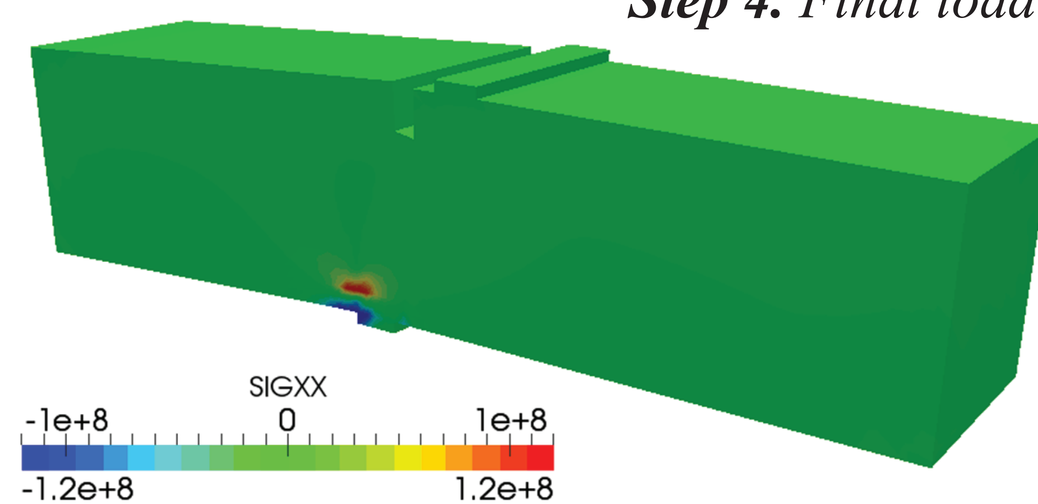
Step 2. *Maximum load*



Step 3. *Crack propagation*



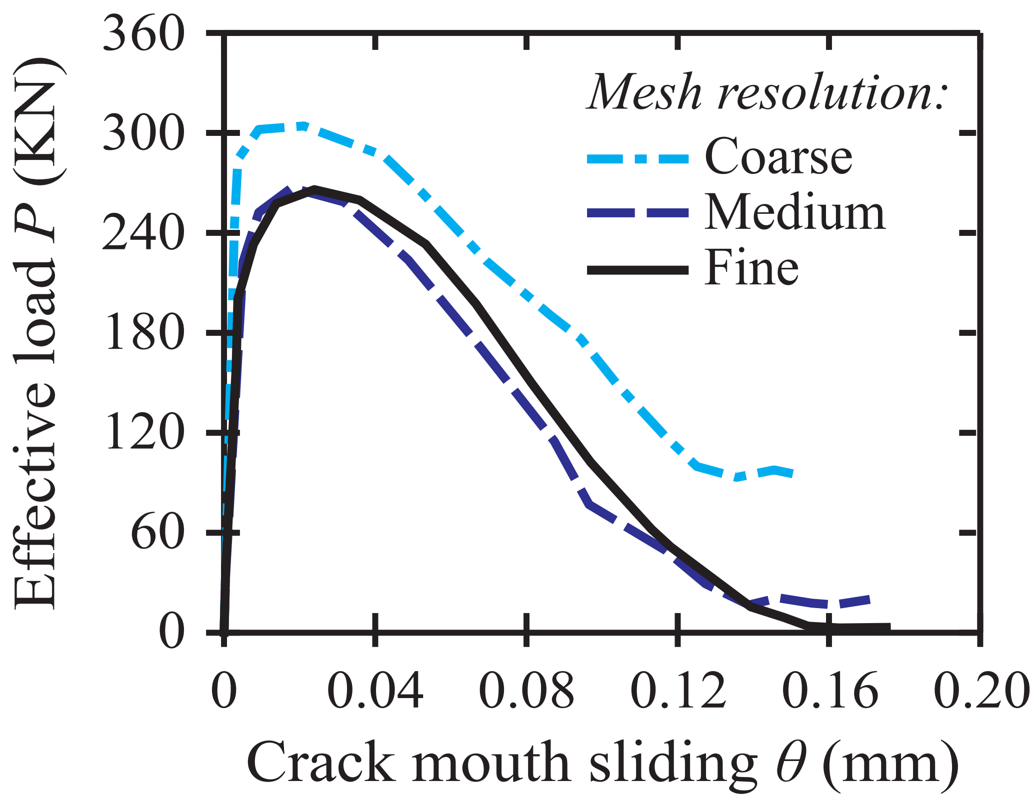
Step 4. *Final loading*

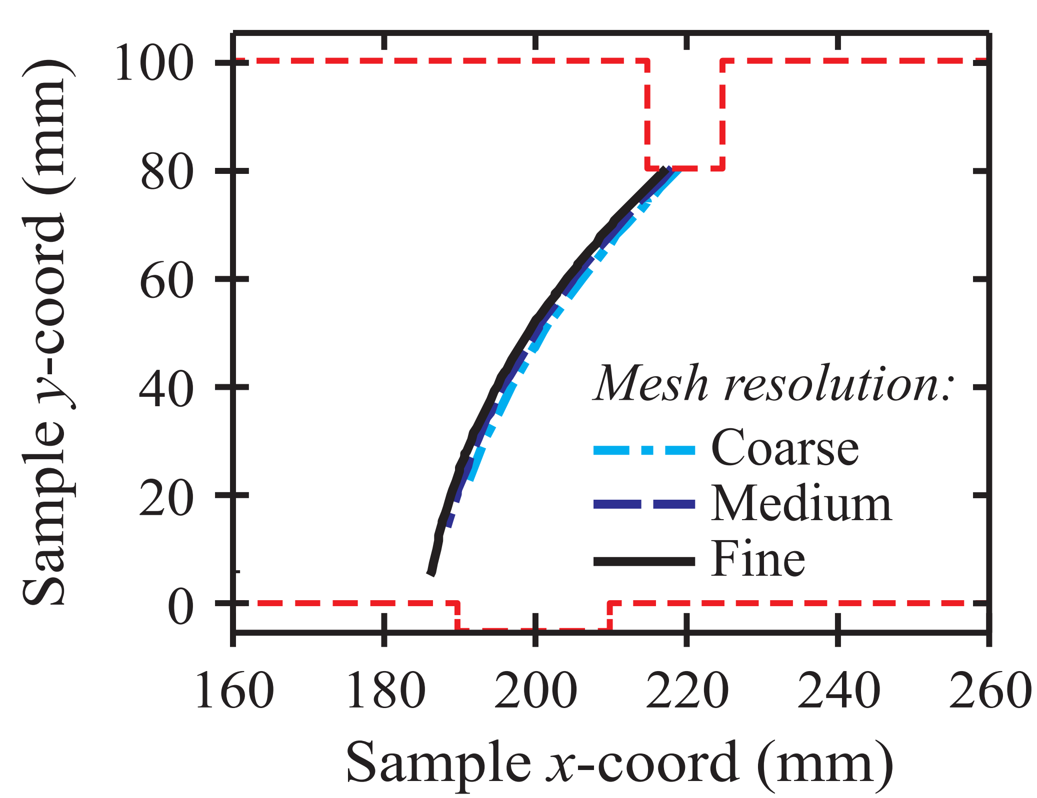


Four points shear test

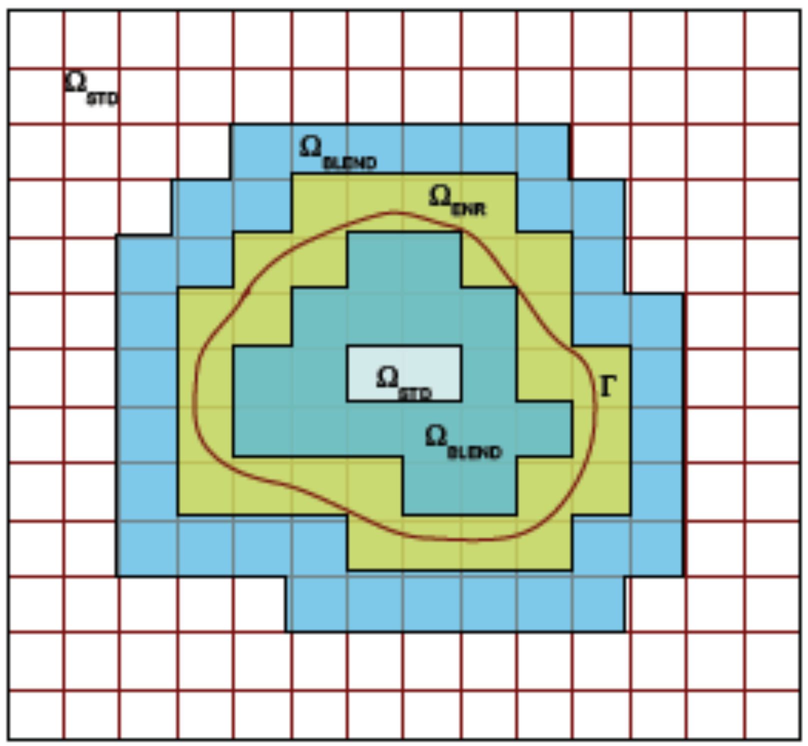
Analysis Mesh Convergence

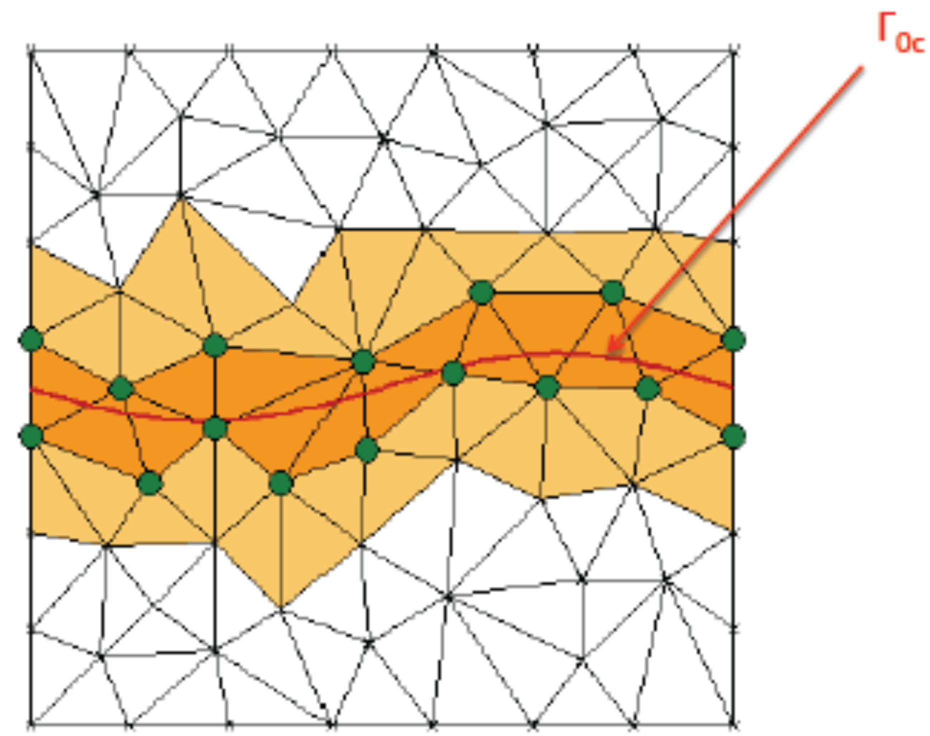
- Coarse mesh: 25.000 elements
- Medium mesh: 200.000 elements
- Fine mesh: 1.600.000 elements





Numerical method: Extended Finite Element Method (XFEM)





- Method adapted to the FE code
- Non-depending mesh
- Crack propagation and simulation heterogeneous materials

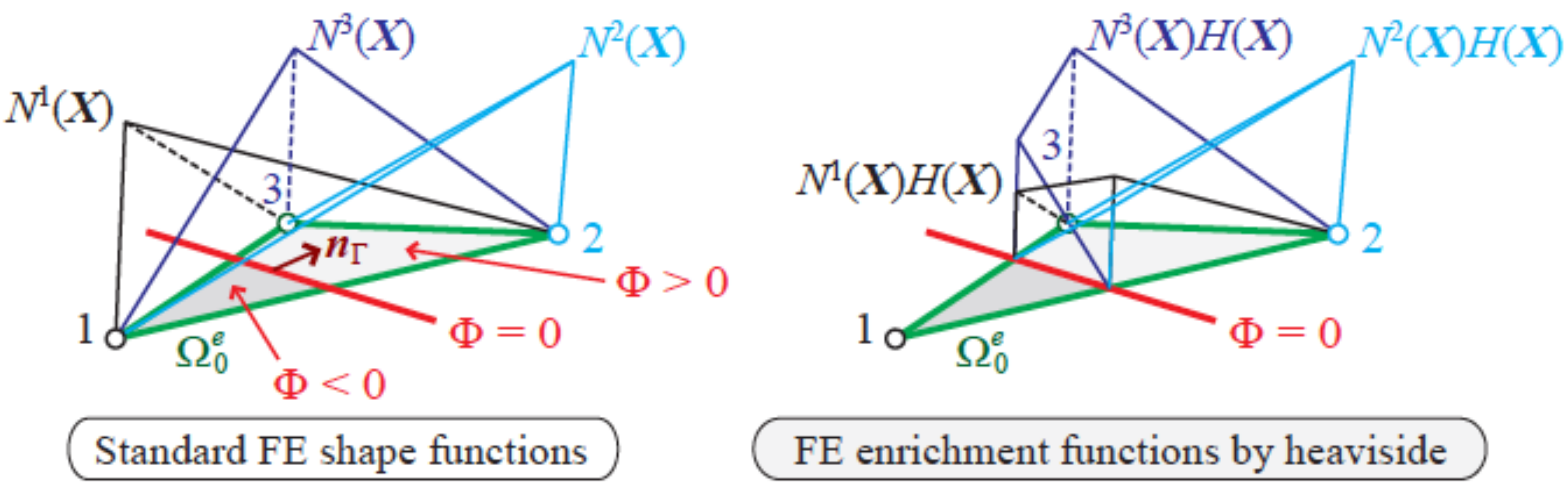
XFEM

- Enrichment of standard FEM solution: approximation to account for discontinuities

$$\boldsymbol{u}^h(\boldsymbol{X}) = \sum_{a \in I} N^a(\boldsymbol{X})\boldsymbol{u}^a \boxed{+ \sum_{a \in I} \tilde{N}^a(\boldsymbol{X})\psi(\boldsymbol{X})\boldsymbol{\alpha}^a}$$

$$\boldsymbol{u}^h(\boldsymbol{X}) = \sum_{a \in I} N^a(\boldsymbol{X})\left[\boldsymbol{u}^a + \{H(\boldsymbol{X}) - H(\boldsymbol{X}^a)\}\boldsymbol{\alpha}^a\right]$$

- Shifted Heaviside functions improve scalability



XFEM

- FEM formulation

$$\int_{B_0} \rho_0 \ddot{\boldsymbol{u}} \cdot \boldsymbol{w} \, \mathrm{d}V_0 + \int_{B_0} \boldsymbol{P} \cdot \boldsymbol{\nabla}_0 \boldsymbol{w} \, \mathrm{d}V_0 = \int_{\Gamma_{0t}} \bar{\boldsymbol{t}}_0 \cdot \boldsymbol{w} \, \mathrm{d}S_0 + \int_{B_0} \boldsymbol{b}_0 \cdot \boldsymbol{w} \, \mathrm{d}V_0$$

- Approximation for the test function

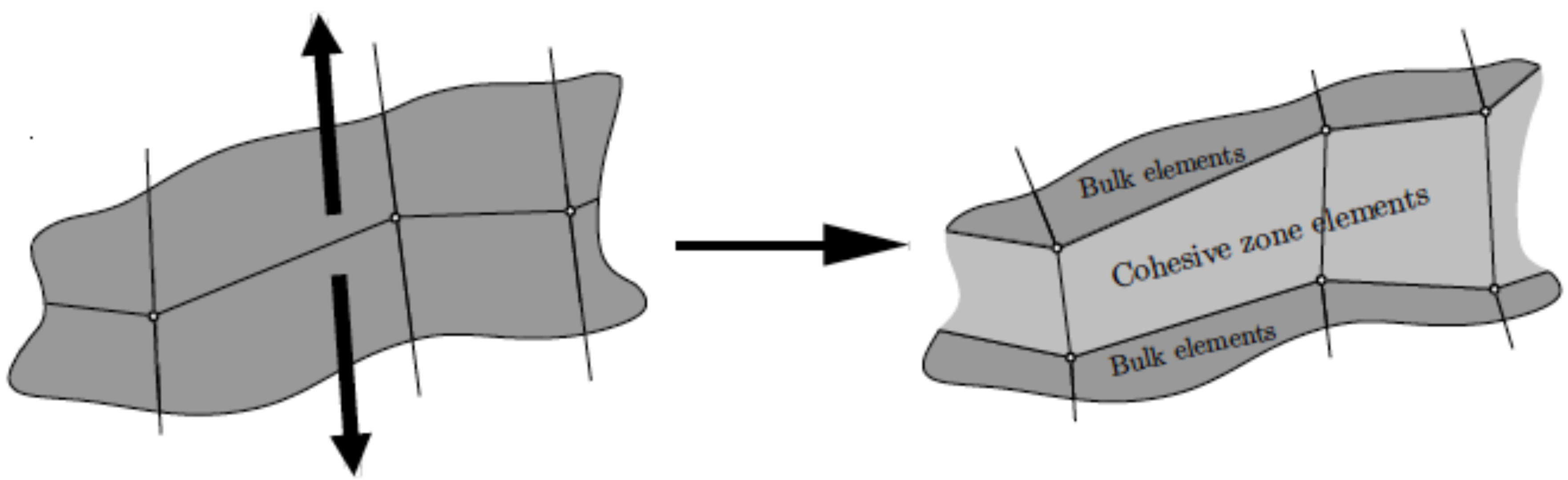
$$\boldsymbol{w}^h(\boldsymbol{X}) = \sum_{b \in I} N^b(\boldsymbol{X}) \boldsymbol{w}^b + \sum_{b \in I_c} N^b(\boldsymbol{X}) \{H(\boldsymbol{X}) - H(\boldsymbol{X}^b)\} \boldsymbol{\beta}^b$$

- Replacing and discretising, in matrix form we obtain the system:

$$\begin{bmatrix} M^{e,ba}_{\boldsymbol{uu}} & M^{e,ba}_{\boldsymbol{u\alpha}} \\ M^{e,ba}_{\boldsymbol{\alpha u}} & M^{e,ba}_{\boldsymbol{\alpha\alpha}} \end{bmatrix} \left\{ \begin{array}{c} \ddot{\boldsymbol{u}}^a \\ \ddot{\boldsymbol{\alpha}}^a \end{array} \right\} + \left\{ \begin{array}{c} \boldsymbol{f}^{e,b}_{\mathrm{int},\boldsymbol{u}} \\ \boldsymbol{f}^{e,b}_{\mathrm{int},\boldsymbol{\alpha}} \end{array} \right\} = \left\{ \begin{array}{c} \boldsymbol{f}^{e,b}_{\mathrm{ext},\boldsymbol{u}} \\ \boldsymbol{f}^{e,b}_{\mathrm{ext},\boldsymbol{\alpha}} \end{array} \right\}$$

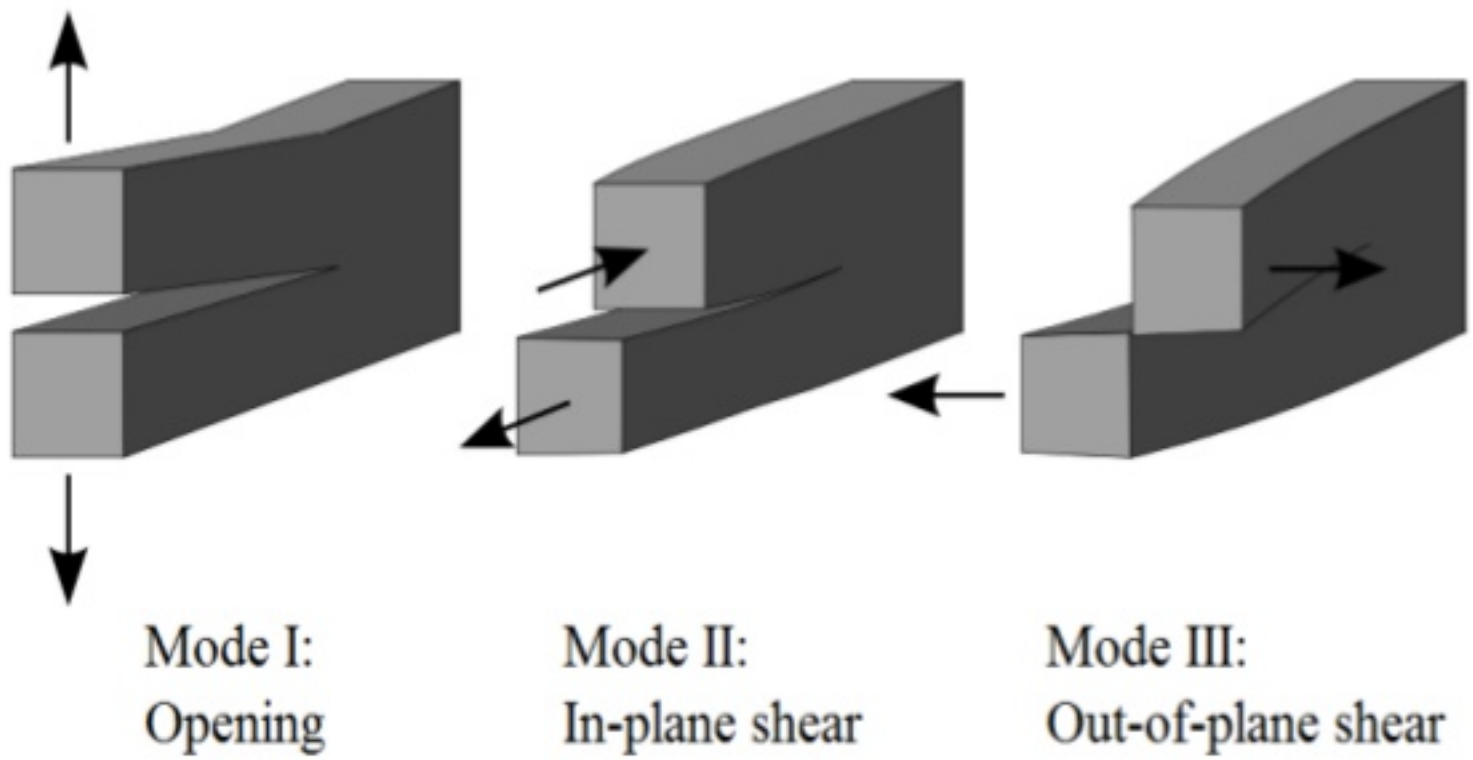
Cohesive zone model

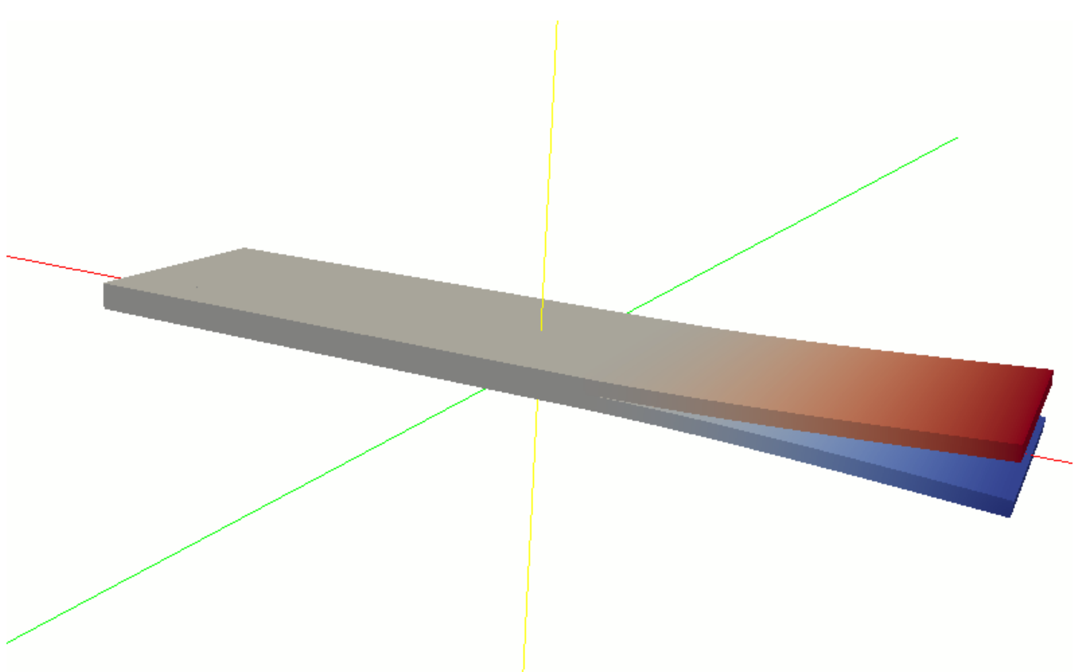
- Cohesive zone elements do not represent any physical material, but describe the cohesive forces which occur when material elements are being pulled apart. Therefore cohesive zone elements are placed between continuum (bulk) elements.
- When damage growth occurs these cohesive zone elements open in order to simulate the crack initiation crack growth.
- The direction of the crack propagation strongly depends on the presence (or absence) of cohesive zone elements (mesh dependence).

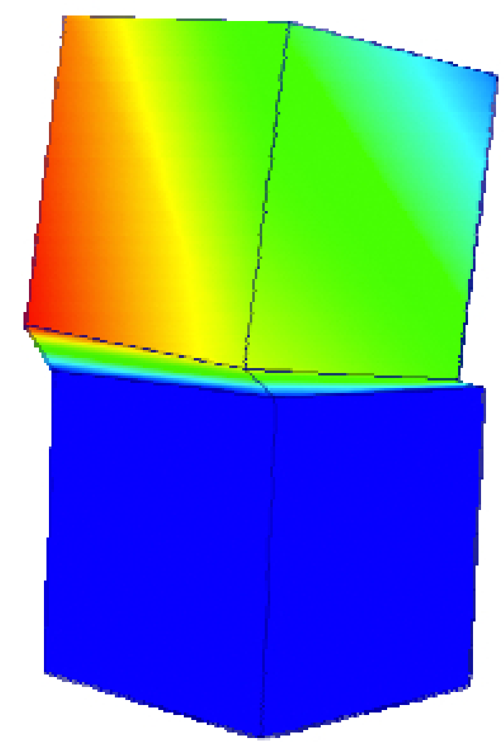


Cohesive zone model

Modes of Failure



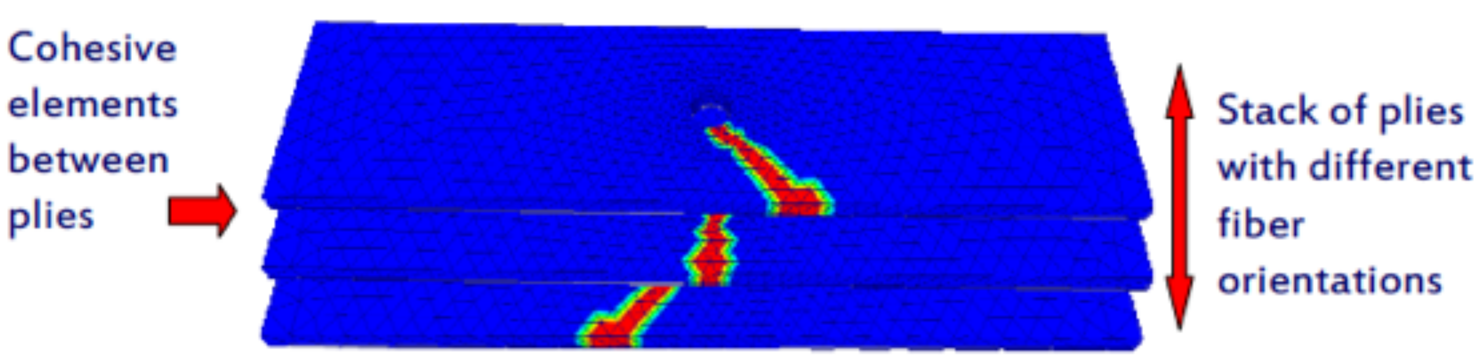


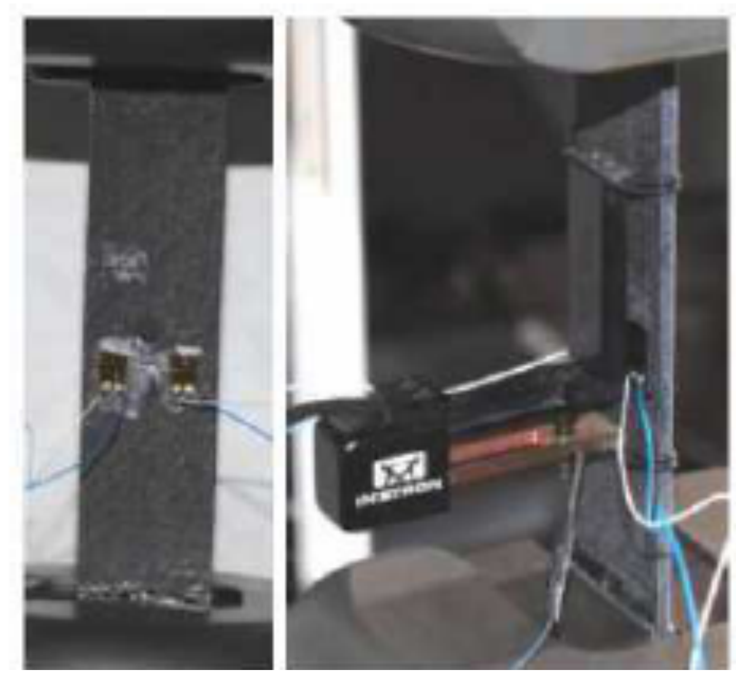


Work done in collaboration with IMDEA materials and Oxford University

Open hole test

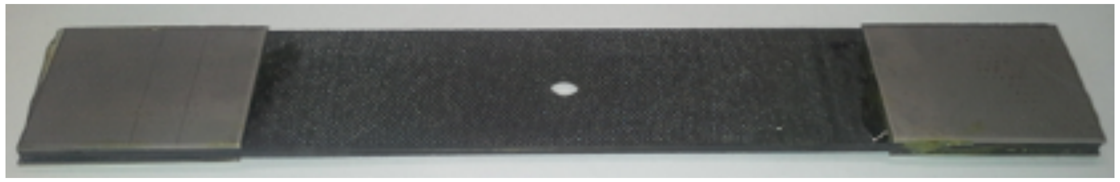
- Carbon fiber/epoxy resin laminate with sequence [90/+45/-45/90/0]s
- Each ply of the laminate has transversely isotropic properties
- Intra-laminar: XFEM + Extrinsic CZM
- Inter-laminar: Intrinsic CZM





Work done by BSC in collaboration with IMDEA materials and Oxford University

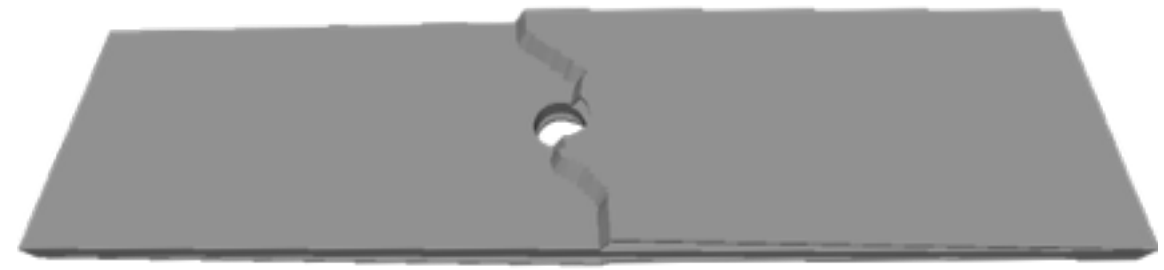
Open hole test



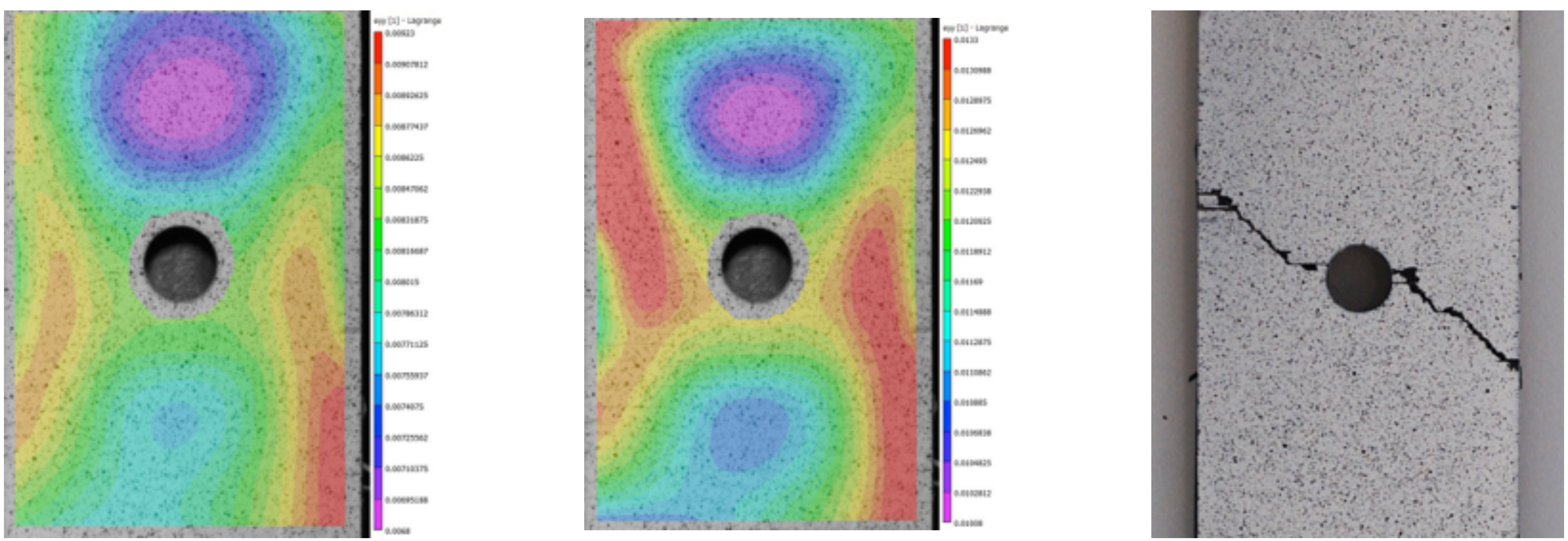
Experimental sample in the initial state

Experimental sample after the tensile test

Numerical model used for simulations



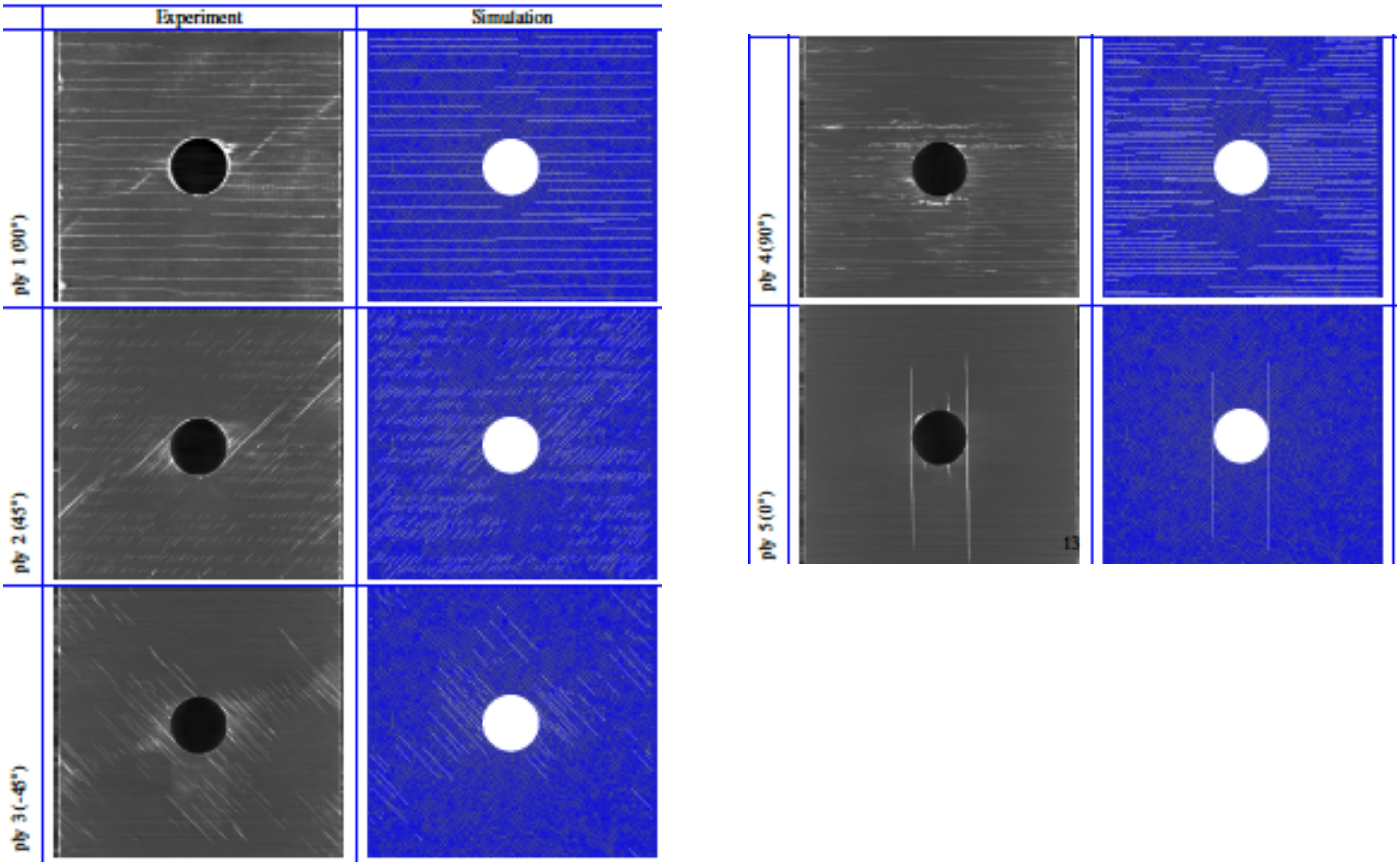
Numerical model after the simulation



Deformation map at 0.8% strain, 1.2% strain and 1.3% (fractured specimen)

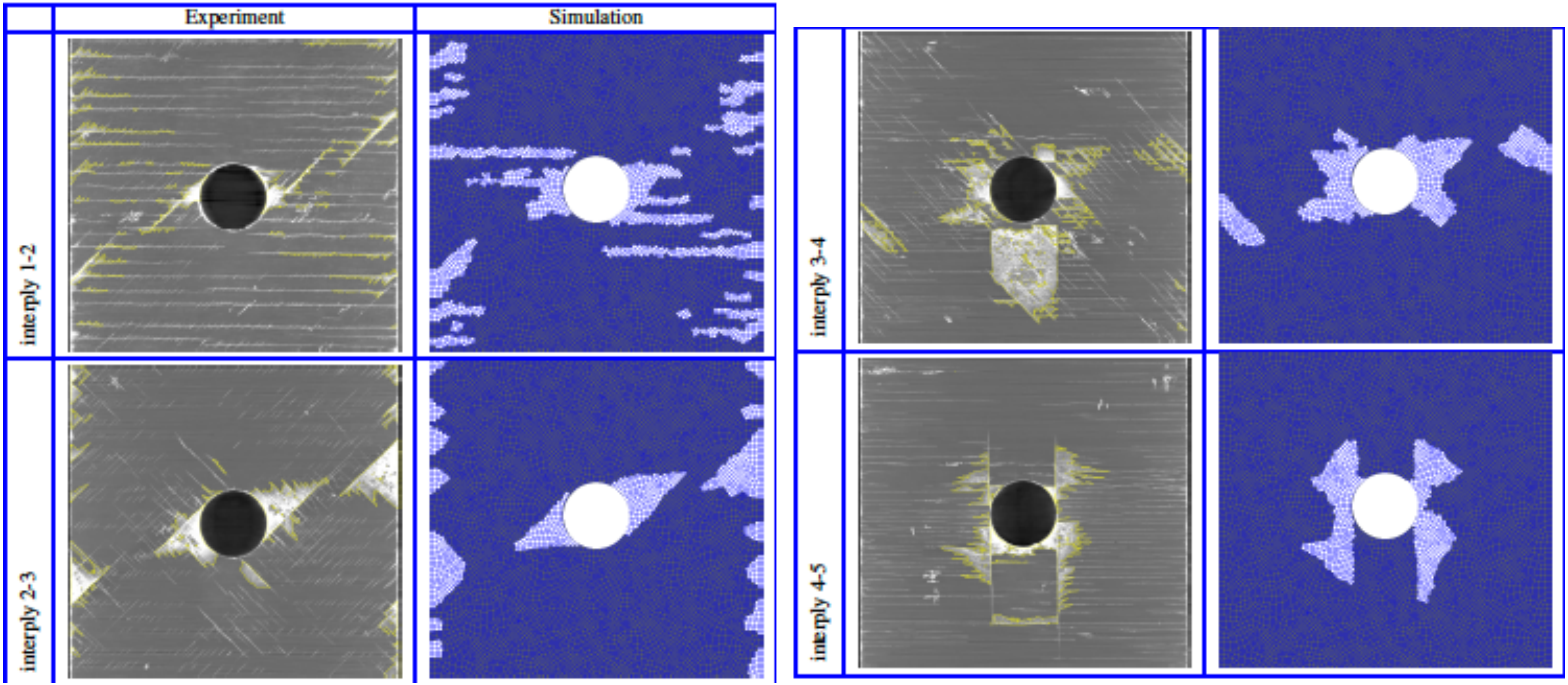
Open hole test

Matrix cracks for plies 1,2,3,4 and 5 at 90% failure load



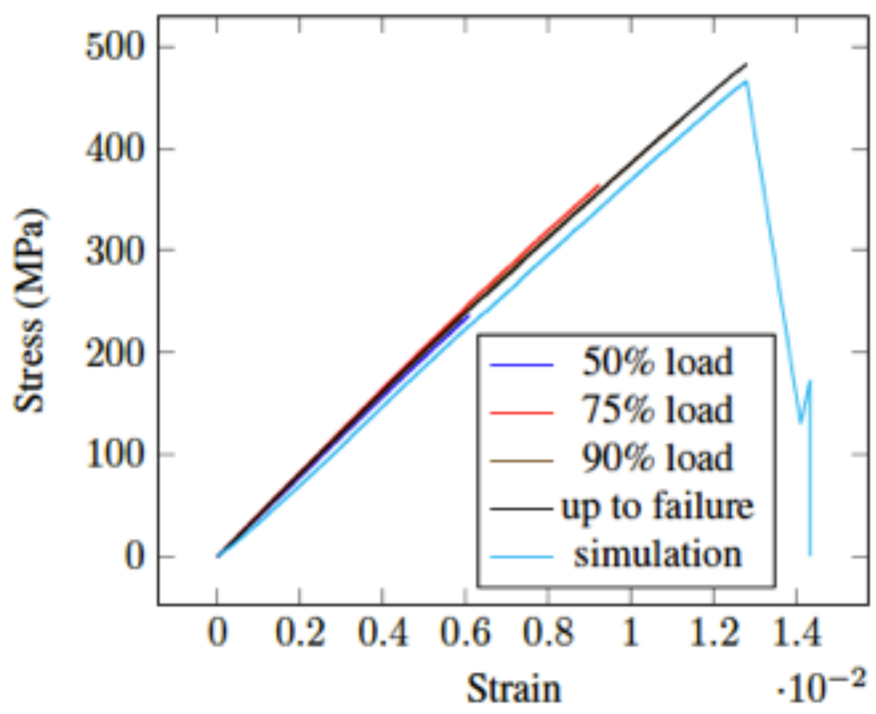
Open hole test

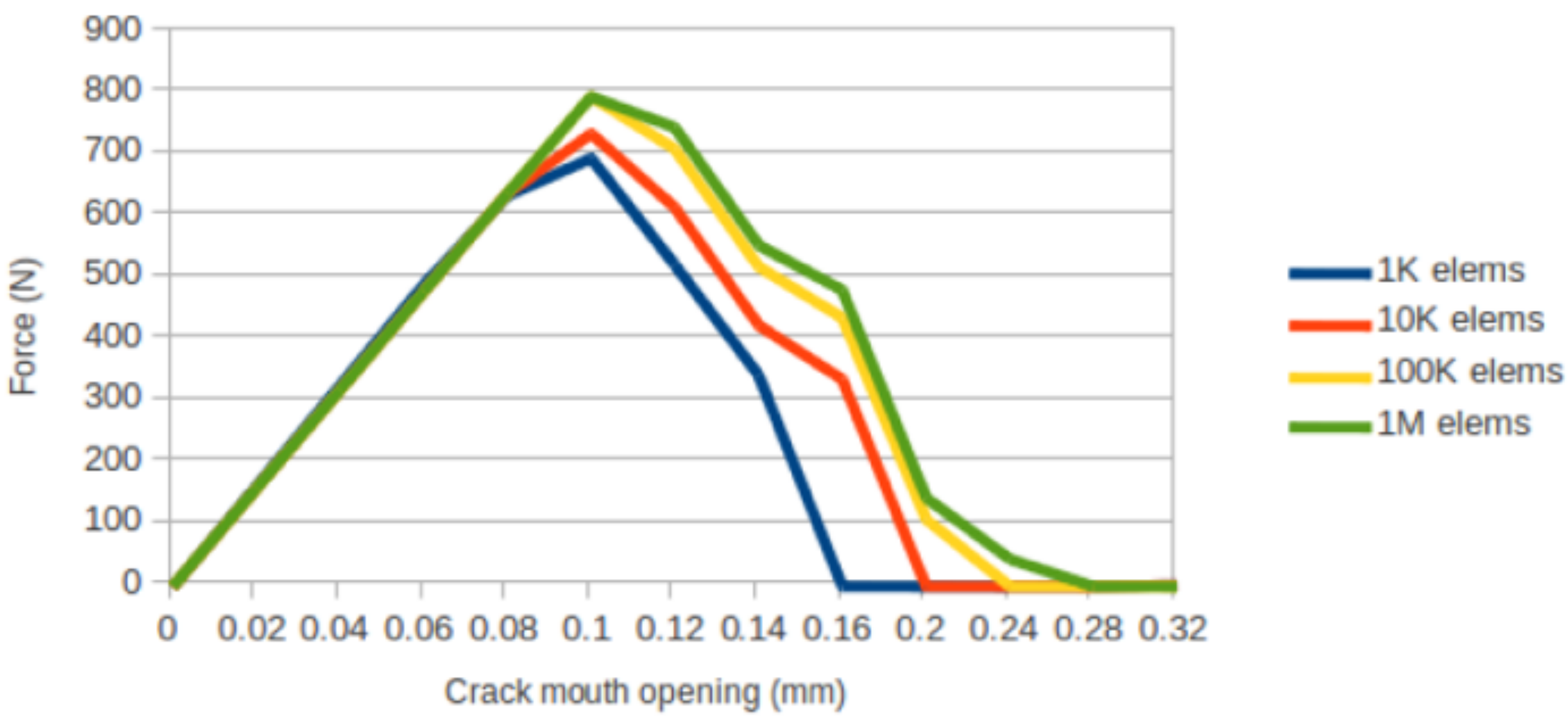
Delamination at 90% failure load



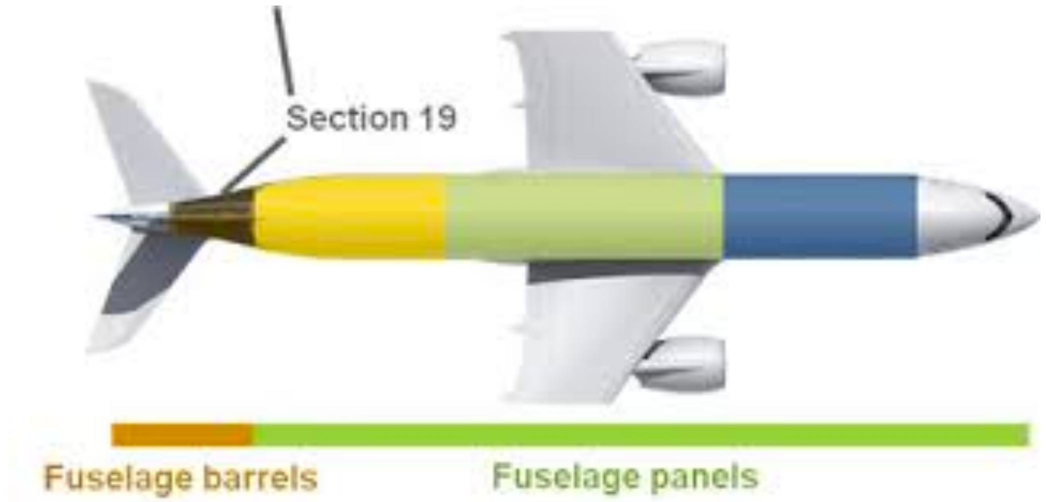
Open hole test

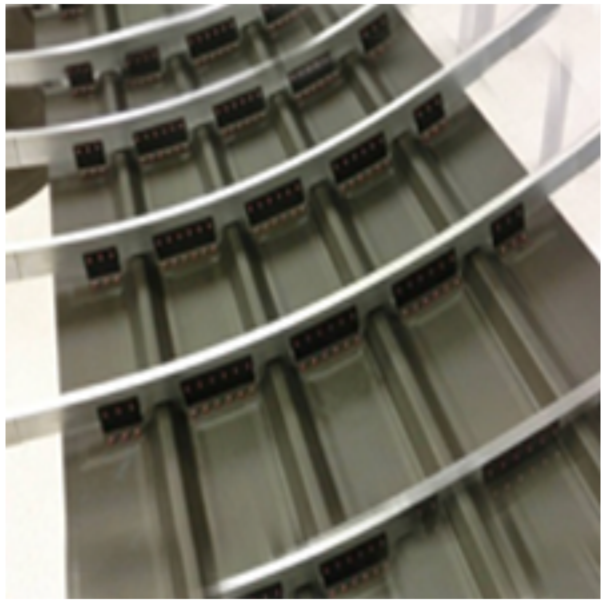
Stress-strain curves for experimental tests and simulation





Composite fuselage panel





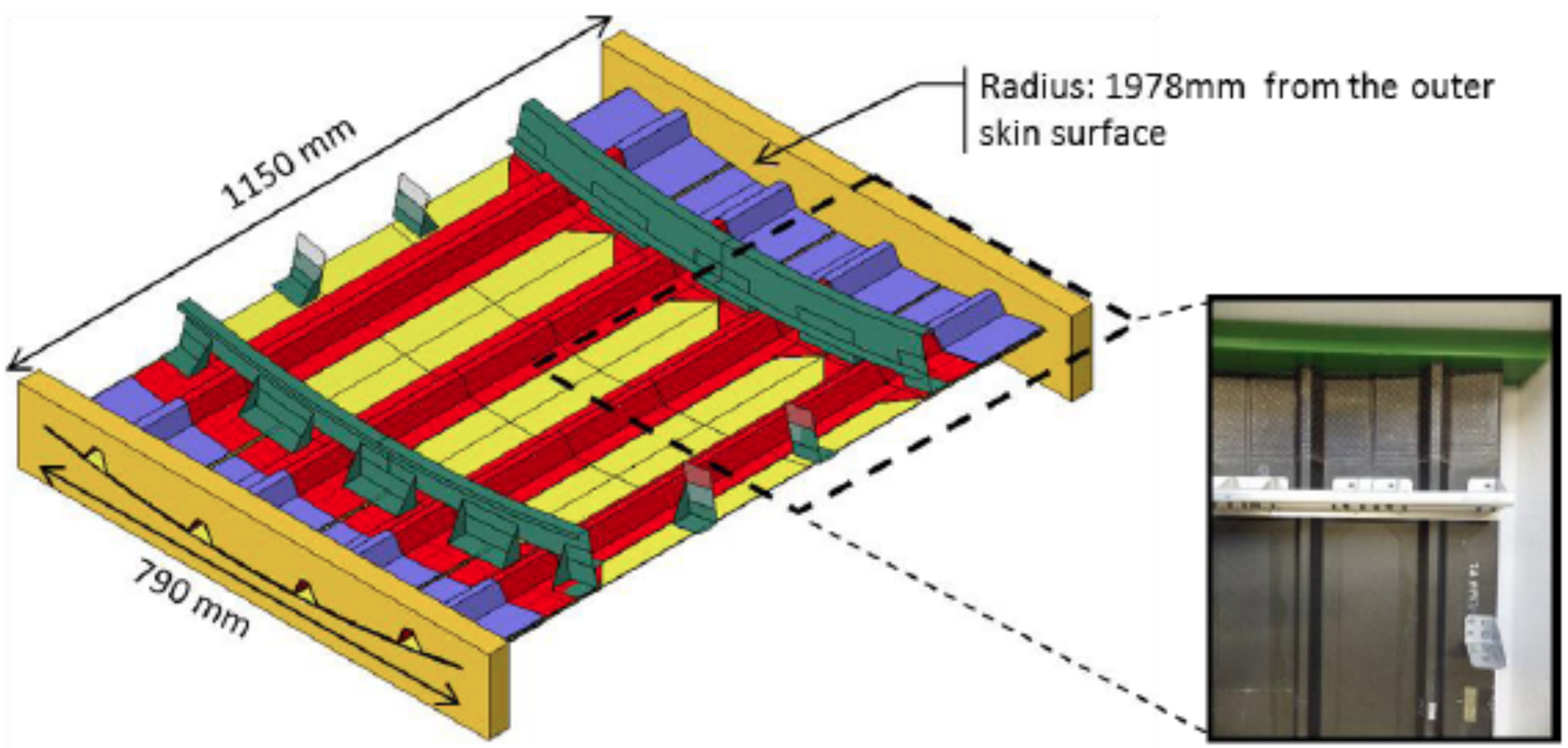
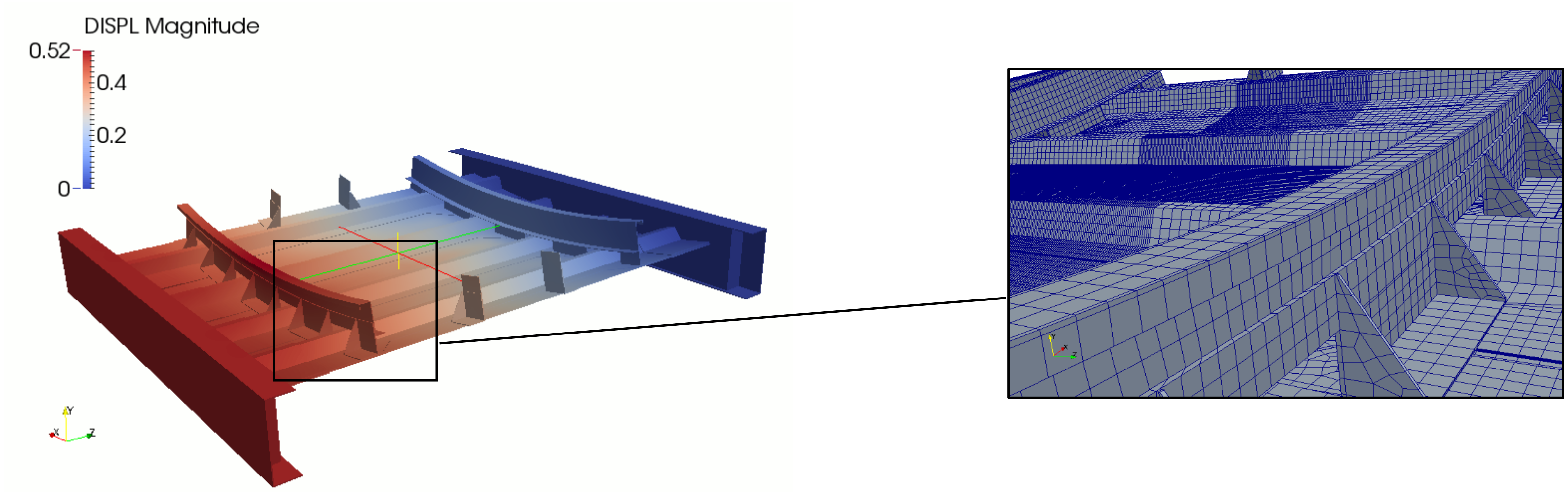
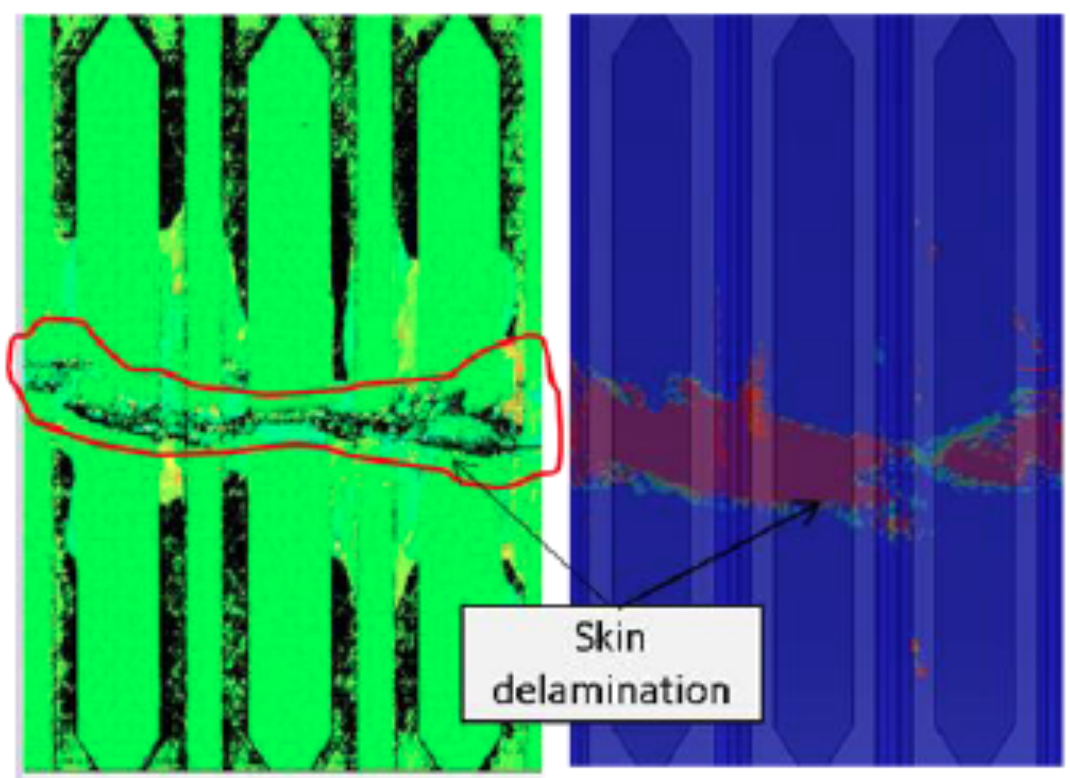
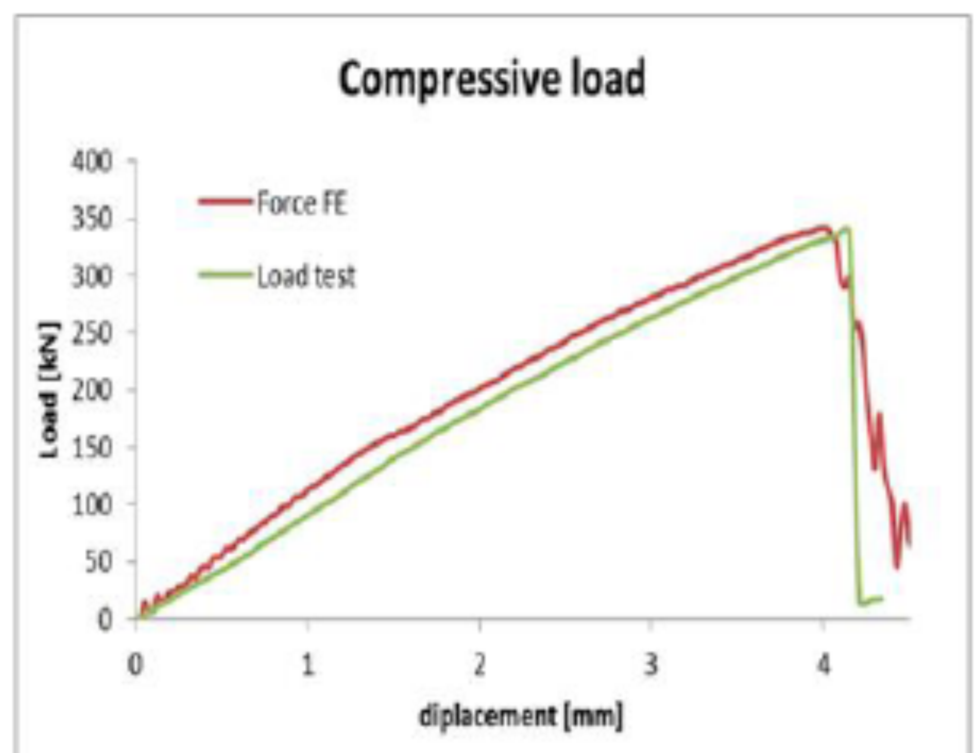


Figure 1 Overview of the panel with its parts; skin (yellow), stiffeners (red), reinforcing plies (blue), potting (gold) and aluminum frames and clips (green-grey).

Composite fuselage panel

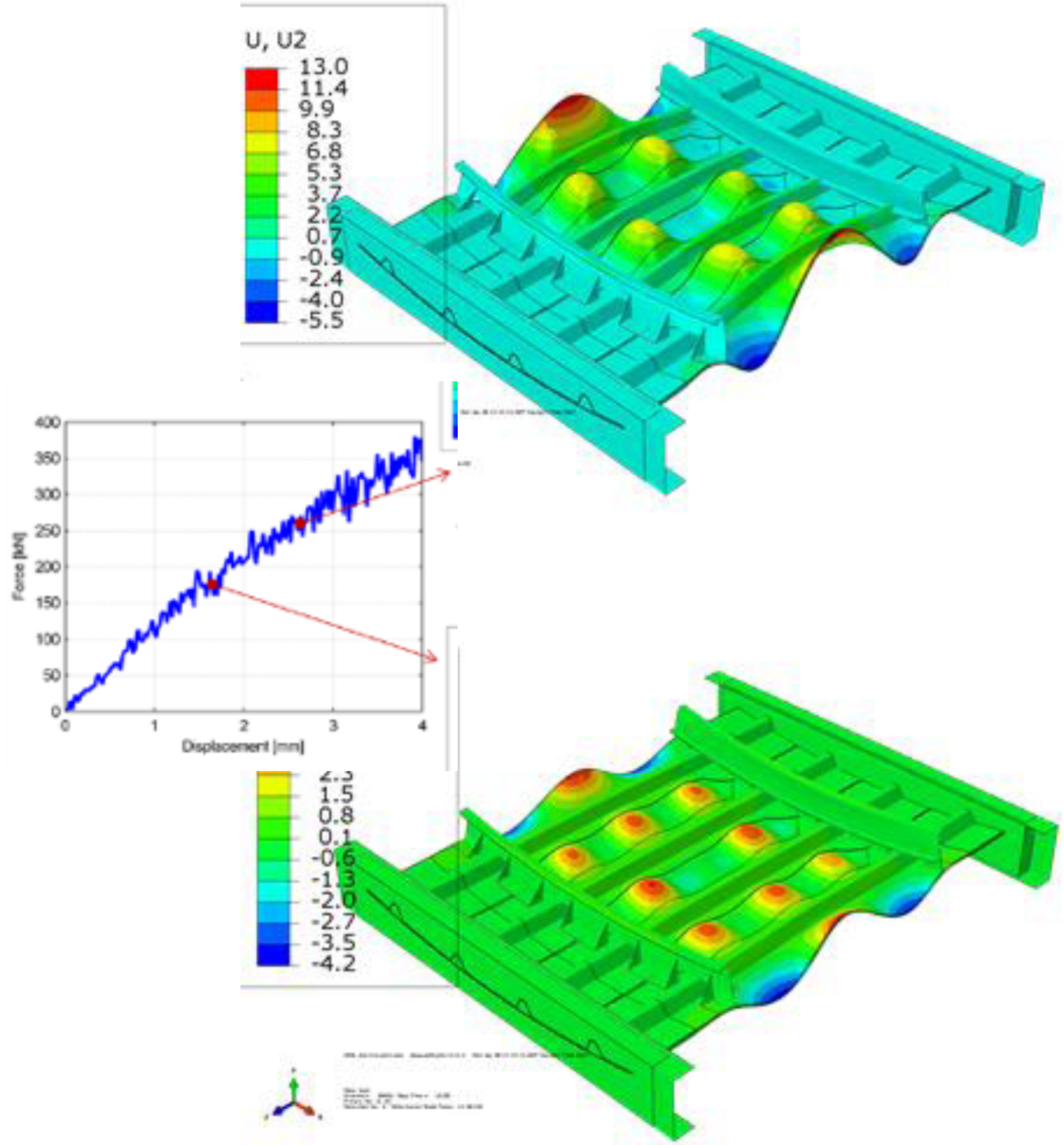






Composite fuselage panel

Fuselage panel under compression after impact test



Force-displacement and out-of-plane deformation of curved fuselage panel

SARISTU project (ICL)

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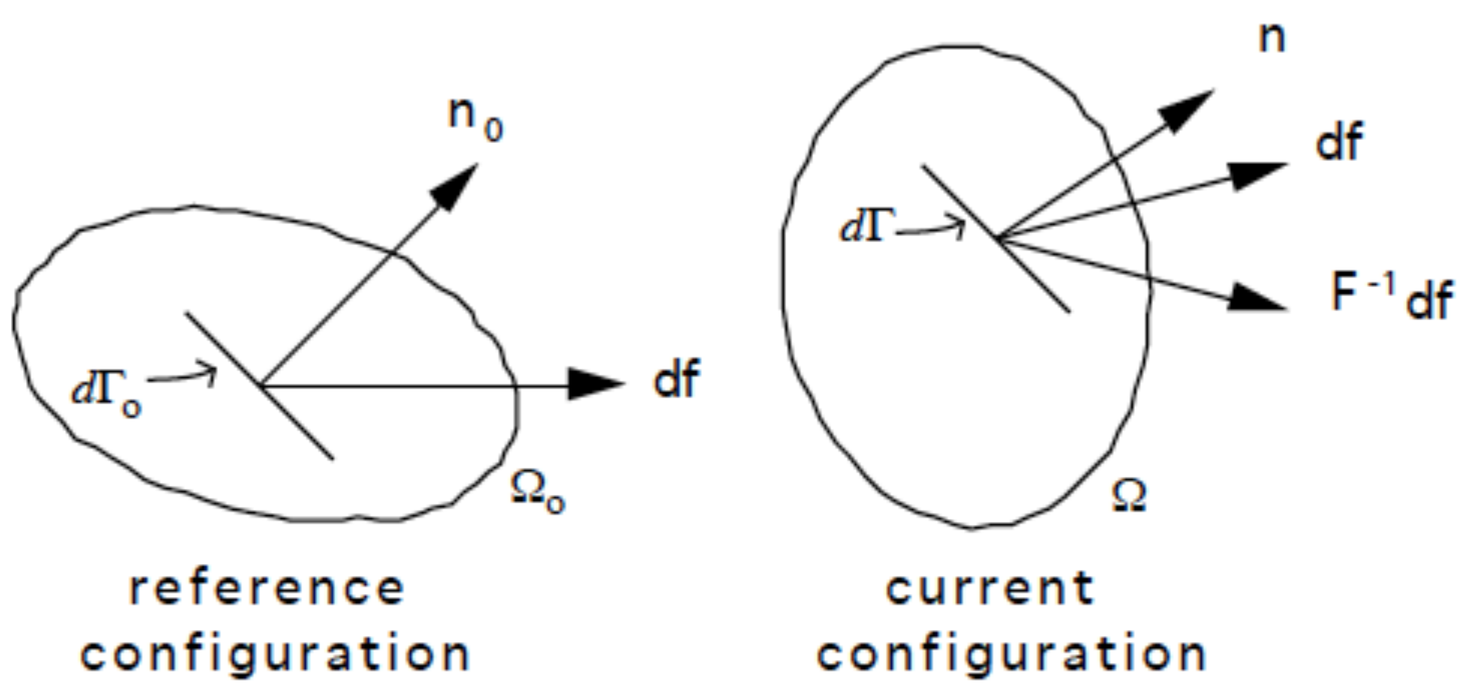
Stress and strain measures

Box 3.1
Definition of Stress Measures



Cauchy stress: $\mathbf{n}\cdot\sigma d\Gamma = d\mathbf{f} = \mathbf{t}d\Gamma$ (3.4.1)

Nominal stress: $\mathbf{n}_0\cdot\mathbf{P}d\Gamma_0 = d\mathbf{f} = \mathbf{t}_0 d\Gamma_0$ (3.4.2)

2nd Piola-Kirchhoff stress: $\mathbf{n}_0\cdot\mathbf{S}d\Gamma_0 = \mathbf{F}^{-1}\cdot d\mathbf{f} = \mathbf{F}^{-1}\cdot\mathbf{t}_0 d\Gamma_0$ (3.4.3)

$d\mathbf{f} = \mathbf{t}d\Gamma = \mathbf{t}_0 d\Gamma_0$ (3.4.4)

For solid mechanics applications, it is instructive to directly develop the conservation equations in terms of the Lagrangian measures of stress and strain in the reference configuration (**P(F)** and **E**).